General Equilbrium Policy Analysis

Lecture 6

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Outline

Auerbach-Kotlikoff Model

Implementation on the PC

Intergenerational distribution

Extension: Earnings-related Pensions

Assumptions

We extend the 'government version' of the Blanchard model to an **Auerbach-Kotlikoff model with mortality risk**.

- Single country, small open economy.
- Endogenous labor supply along intensive and extensive margin.
- **Exogenous effective retirement age**.
- Households walk through $A+1$ **different age groups** (0 to A)...
- ... **unless they die** before at per-period rate $1-\gamma_t^a$.
- ~~Households of same age insure themselves against the risk of longevity.~~
- Unintentionally left-over assets are distributed among the surviving households (**accidental bequest** *ab*).
- Already during life older households transfer assets to younger ones (exogenously, non-microfounded) (**inter-vivo transfers** *iv*).
- Next to the already used government instruments there is a **pay-as-you-go (PAYG) pension system**.
- Other assumptions like before.

Demographics

Let $a \in \{0, 1, ..., A\}$ denote the age group. The mass of **households** of age a evolves according to

$$N^{a+1}_{t+1} = \gamma^{a}_{t+1} N^{a}_{t}, \ 0 \leq a < A, \text{ with } \gamma^{A}_{t+1} = 0,$$
$$N^{0}_{t+1} = NB_{t+1}$$

Life-span is **stochastic** with **expected age** for persons 'born' at t

$$\sum_{a=1}^{A} a(1 - \gamma^{a}_{t+a}) \prod_{s=0}^{a-1} \gamma^{s}_{t+s}.$$

Total population size is simply $N_t = \sum_{a=0}^{A} N^{a}_{t}$.

Notation: For some variable X, X^{a}_{t} refers to the **per-capita term** in age-class a. **Aggregation** is then simply

$$X_t = \sum_{a=0}^{A} X^{a}_{t} N^{a}_{t}.$$

Government instruments

We model the following **fiscal instruments** of the government and **differentiate** between **labor taxes** and **pension contributions** (for workers and firms)

- Total labor tax burden for workers: $\tau^{W,a} = \tau^{W,i,a} + \tau^{W,c,a} - \nu \cdot \tau^{W,i,a} \cdot \tau^{W,c,a}$
- Total labor tax burden for workers: $\tau^{F} = \tau^{F,i} + \tau^{F,c}$
- Lump-sum taxes/transfers from/to households: τ^{l}
- Profit taxes from firms: τ^{prof}
- Profit tax deductibility options for capital maintenance costs: ϕ_0^{τ}
- Consumption taxes: τ^{C}
- Unproductive (non-microfounded) government consumption: C^{G}
- Benefits for non-participating households: b^{a}
- Gross pension payment: P^{a}
- The effective retirement age: reflected in indicator for not being retired ϕ^{a}

Household problem

The **household problem** is

$$V(A_t^a) = \max_{C_t^a, \ell_t^a} \ {}^{1}/_{\rho} \, (Q_t^a)^\rho + \beta \gamma_{t+1}^a G^\rho V(A_{t+1}^{a+1}), \qquad \text{s.t.}$$

$$GA_{t+1}^{a+1} = R_{t+1} \left[A_t^a + \bar{y}_t^a - pc_t C_t^a \right], \quad \bar{y}_t^a = y_t^a + iv_t^a + ab_t^a$$

$$y_t^a = \phi_t^a \left[\delta_t^a (1 - \tau_t^{W,a}) w_t \ell_t^a \theta_t^a + (1 - \delta_t^a) b_t^a \right]$$

$$+ (1 - \phi_t^a)(1 - \tau^{W,i,a}) P_t^a - \tau_t^{l,a}$$

$$Q_t^a = C_t^a - \Psi_t^a, \quad \Psi_t^a = \phi_t^a \left[\delta_t^a \varphi^a(\ell_t^a) + (1 - \delta_t^a) h_t^{a,e} \right]$$

The **behavior describing** equations are

$$\delta_t^a : \quad \left[(1 - \tau_t^{W,a}) w_t \ell_t^a \theta_t^a - b_t^a \right] / pc_t - \varphi^a(\ell_t^a) = \underline{h}_t^a$$

$$\ell_t^a : \quad \varphi^{a\prime}(\ell_t^a) \cdot pc_t = (1 - \tau_t^{W,a}) w_t \theta_t^a$$

$$C_t^a : \quad GQ_{t+1}^{a+1} = \left(\gamma_{t+1}^a \beta R_{t+1} \frac{pc_t}{pc_{t+1}} \right)^\sigma Q_t^a$$

Accidental Bequest

Timing: At the **beginning of the period** households receive accidental bequest (like other income flows). At the end households can die and if they do they leave their savings

$$S_t^a = A_t^a + y_t^a + iv_t^a + ab_t^a - pc_t C_t^a.$$

Total assets **collected**: $\sum_{a=0}^{A}(1-\gamma_{t+1}^a)S_t^a N_t^a$.
Total assets **to distribute**: $\sum_{a=0}^{A} ab_t^a N_t^a$.

We distribute the collected assets according to the following **rule**

$$ab_t^a = \xi_t^a \cdot \frac{\sum_{a=0}^{A}(1-\gamma_{t+1}^a)S_t^a N_t^a}{N_t^a}, \quad \sum_{a=0}^{A}\xi_t^a = 1.$$

Aggregation

The **consumption function** is given by

$$Q_t^a = (\Omega_t^a pc_t)^{-1} \left[A_t^a + H_t^a\right],$$
$$\Omega_t^a = 1 + \left(\gamma_{t+1}^a \beta\right)^\sigma \left(R_{t+1} \frac{pc_t}{pc_{t+1}}\right)^{\sigma-1} \Omega_{t+1}^{a+1}.$$

As also the marginal propensity to consume differs per age-group there is **no way of analytical aggregation**!

In contrast to the Blanchard model we have to **numerically solve** $A+1$ **household** problems instead of just one. We can only **aggregate ex-post**.

Total **labor supply** is $L_t^S = \sum_{a=0}^{A} \phi_t^a \delta_t^a \ell_t^a \theta_t^a N_t^a$. We can easily define average tax or contribution rates, e.g. $\tau_t^W = \frac{\sum_{a=0}^{A} \tau_t^{W,a} \phi_t^a \delta_t^a \ell_t^a \theta_t^a N_t^a}{L_t^S}$.

The Government Sector

Total per-period **revenue** and **expenditure** of the government are

$$Rev_t = T_t^F + \left(\tau_t^F L_t^D + \tau_t^W L_t^S\right) w_t + \tau_t^I N_t + \tau_t^C C_t + \sum_{a=0}^{A} (1 - \phi_t^a) \tau_t^{W,i,a} P_t^a N_t^a,$$

$$Exp_t = C_t^G + \sum_{a=0}^{A} \phi_t^a (1 - \delta_t^a) b_t^a N_t^a + \sum_{a=0}^{A} (1 - \phi_t^a) P_t^a N_t^a.$$

The pension system is included in those flows and can separately be written as

$$Rev_t^P = \left(\tau_t^{F,c} L_t^D + \tau_t^{W,c} L_t^S\right) w_t$$

$$Exp_t^P = \sum_{a=0}^{A} (1 - \phi_t^a) P_t^a N_t^a.$$

If $Rev_t^P < Exp_t^P$ the pension system is implicitly cross subsidized by other taxes apart from the pension contributions.

Temporary Equilibrium

In period t we know

- all parameters and taxes (except for one)
- **predetermined** variables: K_t, D_t^F, D_t^G.
- guesses for **forward looking** variables: V_{t+1}, H_{t+1}^a, and Ω_{t+1}^a $\forall a$.

Three markets have to clear: Labor $(L_t^D = L_t^S)$, assets $(A_t = V_t + D_t^F + D_t^G)$ and goods $(Y_t = C_t + I_t + J_t + C_t^G + TB_t)$.

Government budget has to hold: $GD_{t+1}^G = R_{t+1}\left(D_t^G - PB_t\right)$ by changing at least one government instrument endogenously.

Respective code is `AuerbachKotlikoff`.

Outline

Implementation on the PC

Foresight variables: H, V and Ω now with much higher dimension: $(2 * (A + 1) + 1) \times T)$, **Predetermined**: K and D^F

`AuerbachKotlikoff` is **not calibrated** to a specific country (task for final project).

- We discuss code implementations: `AuerbachKotlikoff`.
- We look at the following
  1. implementation
  2. briefly calibration
  3. demographic shock
  4. standard reforms
  5. store guesses
  6. computation speed

  and interpret the results.

Outline

Measuring intergenerational redistribution

- What is a **generation** in our implementation?
- **Compare indirect (life-time) utilities** (V_t^a) of different age groups at different points in time.
- Change versus the ISS can be translated in **consumption equivalent terms**, i.e. by which constant factor do you have to multiply ISS-consumption for all remaining age periods to end up with same indirect utility.

Implemented in routines `welfare` (change in indirect utility) and `welfareC` (change in consumption equivalent terms).

The functions are available in `AuerbachKotlikoff` and `AuerbachKotlikoff_earnings_link`.

Outline

Earnings-related Pensions 1/2

We explicitly model the **link between labor income and pension payouts** by introducing another **stock pension entitlements** P_t^a. The law of motion is

$$GP_{t+1}^{a+1} = G_t^{P,a}\left[P_t^a + m_t^a \phi_t^a \delta_t^a w_t \ell_t^a \theta_t^a\right].$$

$G_t^{P,a}$ captures the **indexation** of pensions (e.g. wage versus price indexation). m_t^a is the **accumulation factor**. Pension payouts enter household income

$$\begin{aligned} y_t^a = \phi_t^a & \left[\delta_t^a(1-\tau_t^{W,a})w_t\ell_t^a\theta_t^a + (1-\delta_t^a)b_t^a\right] \\ & + (1-\phi_t^a)\,(1-\tau^{W,i,a})\left[\varsigma_t^a P_t^a + P_t^0\right] - \tau_t^{l,a}, \end{aligned}$$

where P_t^0 is a **flat pension** part (age- and earnings-independent). ς_t^a is used to cut or raise **earnings-related pension** payouts.

Earnings-related Pensions 2/2

Household utility now depends on **two stocks**, i.e. $V(A_t^a, P_t^a)$. The two envelope conditions are

$$A_t^a: \;\; \lambda_t^a = \gamma_{t+1}^a \beta G^{\rho-1} R_{t+1} \lambda_{t+1}^{a+1}$$

$$P_t^a: \;\; \eta_t^a = \gamma_{t+1}^a \beta G^{\rho-1} \left[G_t^{P,a} \eta_{t+1}^{a+1} + R_{t+1} \lambda_{t+1}^{a+1} (1 - \tau_t^{W,a}) \varsigma_t^a (1 - \phi_t^a) \right]$$

Rewriting the **optimality conditions** for hours supply and participation implies the following changes to the **effective tax rates**

$$\hat{\tau}_t^{W,a} = \frac{\tau_t^C + \tau_t^{W,a} - m_t^a \frac{G_t^{P,a}}{R_{t+1}} \frac{\eta_{t+1}^{a+1}}{\lambda_{t+1}^{a+1}}}{1 + \tau_t^C}.$$

$$\hat{\tau}_t^{\delta,a} = \frac{\tau_t^C + \tau_t^{W,a} + b_t^a/(w_t \ell_t^a \theta_t^a) - m_t^a \frac{G_t^{P,a}}{R_{t+1}} \frac{\eta_{t+1}^{a+1}}{\lambda_{t+1}^{a+1}}}{1 + \tau_t^C}.$$

The earnings-link implies that pension contributions are **not fully perceived as taxes**.

Temporary Equilibrium

Define $\tilde{\lambda}^a_{t+1} = \lambda^a_{t+1}/\eta^a_{t+1}$.

In period t we know

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- guesses for **forward looking** variables: V_{t+1}, H^a_{t+1}, Ω^a_{t+1} and $\tilde{\lambda}^a_{t+1}$ $\forall a$.

Three markets have to clear: Labor ($L^D_t = L^S_t$), assets ($A_t = V_t + D^F_t + D^G_t$) and goods ($Y_t = C_t + I_t + J_t + C^G_t + TB_t$).

Government budget has to hold: $GD^G_{t+1} = R_{t+1}\left(D^G_t - PB_t\right)$ by changing at least one government instrument endogenously.

Respective code is `AuerbachKotlikoff_earnings_link`.