General Equilbrium Policy Analysis

Lecture 4

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Outline

Homework 2

Effective taxation, marginal and average tax rate and progressivity

Detrending

Mixture of anticipated and unanticipated shocks

Walras' Law

Solution to Ex. 1

Make use of the fact that death and aging shocks are **memoryless within an age class**.

The remaining life expectancy of the last age class $\Theta(A)$ in analogy to the Blanchard model is

$$\Theta^A = \gamma^A/(1-\gamma^A). \tag{1.1}$$

Then we work backwards and express conditional life expectancy recursively as

$$\Theta^a = \frac{\gamma^a\omega^a + (1-\omega^a)\gamma^a(\Theta^{a+1}+1)}{1-\gamma^a\omega^a}. \tag{1.2}$$

Life expectancy at birth is equal to Θ^1.

Show: Some simulations using the code `simlifeexpect`.

Outline

Taxation at the intensive and the extensive margin

Recall the **two conditions** for the intensive margin (ℓ) and extensive margin (δ)

$$\delta_t : \quad \left[(1-\tau_t^W)w_t\ell_t\theta_t - b_t\right]/pc_t - \varphi(\ell_t) = \underline{h}_t$$

$$\ell_t : \quad \varphi'(\ell_t)\cdot pc_t = (1-\tau_t^W)w_t\theta_t$$

The government instruments τ^W, b and τ^C distort these decisions. A way to summarize the total distortion $\Rightarrow$ **effective tax rates**.

Effective taxe rates

The **total distortion** at a margin can be summarized by a **single effective tax rate**.

The **intensive margin**:

$$\varphi'(\ell_t) = (1 - \hat{\tau}_t^W) w_t \theta_t, \quad \hat{\tau}_t^W = \frac{\tau_t^C + \tau_t^W}{1 + \tau_t^C}$$

The **extensive margin** (participation tax):

$$(1 - \hat{\tau}_t^\delta) w_t \ell_t \theta_t - \varphi(\ell_t) = \underline{h}_t, \quad \hat{\tau}_t^\delta = \frac{\tau_t^C + \tau_t^W + b_t/(w_t \ell_t \theta_t)}{1 + \tau_t^C}.$$

Average vs. marginal tax rates

So far we assumed a **linear tax schedule**, e.g. tax liability is $\tau^W \cdot w\ell\theta$. But what if this is not the case?

Assume as a **generalization** that the tax liability is $T(w\ell\theta)$. Then the two labor supply optimality conditions are:

$$\delta_t : \quad \left[\left(1 - \frac{T^W(w_t\ell_t\theta_t)}{w_t\ell_t\theta_t}\right) \cdot w_t\ell_t\theta_t - b_t\right] / pc_t - \varphi(\ell_t) = \underline{h}_t$$

$$\ell_t : \quad \varphi'(\ell_t) \cdot pc_t = \left(1 - T^{W\prime}(w_t\ell_t\theta_t)\right) \cdot w_t\theta_t$$

where $\frac{T^W(w_t\ell_t\theta_t)}{w_t\ell_t\theta_t}$ is the **average tax rate** and $T^{W\prime}(w_t\ell_t\theta_t)$ is the **marginal tax rate**. In case of a linear tax schedule, i.e. $T^W(w\ell\theta) = \tau^W \cdot w\ell\theta$ both are equal to τ^W.

Measures of progressivity

Definition (Progressive taxation)

First, define earnings $e = w\ell\theta$. A tax schedule $T^W(e)$ is progressive (regressive) iff the average tax rate is increasing (decreasing) in income e.

As $d(T^W(e)/(e))/de = \frac{T^{W\prime}(e) - T^W(e)/e}{e}$ it is clear that if a schedule is progressive (regressive) the marginal rate will exceed (fall short of) the average rate, i.e. $T^{W\prime}(e) > (<) T^W(e)/e$.

Two measures of **local progressivity**:

- Coefficient of **tax liability progression**: defined as elasticity of T w.r.t. e, i.e.

$$\varepsilon = \frac{dT^W}{de}\frac{e}{T^W} = \frac{\text{marg. rate}}{\text{aver. rate}}. \tag{2.1}$$

- Coefficient of **residual income progression**: defined as elasticity of $e - T$ w.r.t. e, i.e.

$$\eta = \frac{de - T^W}{de}\frac{e}{e - T^W} = \frac{1 - \text{marg. rate}}{1 - \text{aver. rate}}. \tag{2.2}$$

Outline

Detrending (1/3)

In our framework we usually use **exogenous growth of labor productivity** by rate g implying a growth factor $G = 1 + g$.

Denote the **level of technological progress** as X then the current level is

$$X_t = G^t X_0. \tag{3.1}$$

In a **balanced growth setting** all variables of the economy (output, capital stock, consumption, investment, ...) grow at rate g we therefore want to detrend by this growth to retain the characteristic of an equilibrium where **all variables are constant**.

Detrending (2/3)

How does **detrending** work?

Denote non-detrended variables using a tilde, e.g. $\tilde{K}_t$. Then the detrended capital stock is

$$K_t = \tilde{K}_t / X_t \quad \Rightarrow \quad \tilde{K}_t = X_t K_t \tag{3.2}$$

We simply use **this definition** in all our equations, e.g. for the law of capital and the consumption function

$$\tilde{K}_{t+1} = (1 - \delta^K)\tilde{K}_t + \tilde{I}_t \quad \Leftrightarrow \quad GK_{t+1} = (1 - \delta^K)K_t + I_t. \tag{3.3}$$

$$\tilde{C}_t = (\Omega_t)^{-1}\left(\tilde{A}_t + \tilde{H}_t\right) \quad \Rightarrow \quad C_t = (\Omega_t)^{-1}(A_t + H_t) \tag{3.4}$$

Hence, in **linear difference equations** G will pop up.

Detrending (3/3)

How to proceed for **non-linear** difference equations, e.g. indirect utility with non-linear felicity, for example $u(x) = x^\rho$?

$$U_t = u(\tilde{C}_t) + \beta U_{t+1}, \quad \Rightarrow \quad U_t = u(X_t C_t) + \beta U_{t+1}. \tag{3.5}$$

Observe that $u(X_t C_t) = (X_t)^\rho (C_t)^\rho$, $u(X_{t+1} C_{t+1}) = (X_t G)^\rho (C_{t+1})^\rho$, etc. Hence, doing a **positive monotone transformation** $V_t = U_t/(X_t)^\rho$ results in the for the maximization problem equivalent expression

$$V_t = u(C_t) + \beta G^\rho V_{t+1}. \tag{3.6}$$

Despite detrending, $g > 0$ has real effects (on **effective discounting**), e.g. for human wealth (here for the Ramsey model)

$$H_t = y_t + \frac{G H_{t+1}}{R_{t+1}} \quad \text{in st.st.} \quad H = y \cdot \frac{1+r}{r-g}. \tag{3.7}$$

Outline

Mixture of anticipated and unanticipated shocks

In principle agents **learn about a reform in** $t = 1$.

Problem: How to simulate two reforms becoming effective at t_1 and $t_2 > t_1$, when agents **cannot anticipate** the second reform and learn about it only in t_2?

We discuss code implementations: `Blanchard_unanticipated`.

Possible applications: Introduction of a pension reform during the ongoing demographic transition.

Outline

Walras' Law

Good modeling discipline: Always check Walras' Law which has to hold even outside of equilibrium.

Define the following **excess demands**

$$\begin{aligned}
\text{assets}: \quad & \zeta_t^A = V_t + D_t^F + D_t^G - A_t \\
\text{labor}: \quad & \zeta_t^L = L_t^D - L_t^S \\
\text{goods}: \quad & \zeta_t^Y = C_t + I_t + J_t + C_t^G + TB_t - Y_t \\
\text{government}: \quad & \zeta_t^G = Rev_t - Exp_t - PB_t
\end{aligned}$$

Then **Walras' Law** is (proof in script)

$$\zeta_t^Y + w_t \zeta_t^L + \zeta_t^A + \zeta_t^G - \frac{G \zeta_{t+1}^A}{R_{t+1}} = 0.$$

Discuss in code `Blanchard_government`.