General Equilbrium Policy Analysis

Lecture 3

Philip Schuster

at the Institute for Advanced Studies, Vienna

Vienna, April 20, 2015

Outline

Intensive Margin - Endogenous Hours Supply 1/3

Assumption: Consumption bundle Q consists of goods consumption and leisure in an **additively separable way** (i.e. **no income effects**). Otherwise aggregation of human wealth H in Blanchard model **not** possible!

$$Q_{v,t} = C_{v,t} - \varphi(\ell_{v,t}),$$

where $\varphi(\cdot)$ is increasing and convex. θ_t is an exogenous labor productivity shift parameter.

The household problem:

$$\begin{aligned} U(A_{v,t}) &= \max_{C_{v,t},\ell_{v,t}} u(Q_{v,t}) + \beta\gamma_{t+1} U(A_{v,t+1}), \quad \text{s.t.} \\ \gamma_{t+1} A_{v,t+1} &= R_{t+1}\left[A_{v,t} + w_t \ell_{v,t}\theta_t - C_{v,t}\right] \quad \text{and} \\ Q_{v,t} &= C_{v,t} - \varphi(\ell_{v,t}). \end{aligned}$$

Intensive Margin - Endogenous Hours Supply 2/3

The **optimality** and **envelope conditions** are

$$\begin{aligned} C_{v,t}: \quad & u'(Q_{v,t}) = \beta R_{t+1}\lambda_{v,t+1}, \\ \ell_{v,t}: \quad & u'(Q_{v,t})\varphi'(\ell_{v,t}) = \beta R_{t+1}\lambda_{v,t+1} w_t \theta_t \\ A_{v,t}: \quad & \lambda_{v,t} = \beta \lambda_{v,t+1} R_{t+1}. \end{aligned}$$

Combining the first two optimality conditions reveals that hours are given by the **simple static relationship**

$$\varphi'(\ell_{v,t}) = w_t\theta_t \quad \Rightarrow \quad \ell_{v,t} = \ell_t.$$

Hence, **aggregate labor supply** is $L_t^S = \ell_t \theta_t N_t$.

Intensive Margin - Endogenous Hours Supply 3/3

The **aggregate household behavior** equations are

$$Q_t = \Omega_t^{-1}(A_t + H_t), \qquad \text{and} \qquad C_t = Q_t + \varphi(\ell_t)N_t,$$

where

$$H_t = w_t L_t^S - \varphi(\ell_t)N_t + \gamma_{t+1}\frac{H_{t+1}(N_t/N_{t+1})}{R_{t+1}}.$$

Firm and market clearing condition are as before.

The **intensive margin** is implemented in `Blanchard_intensive`.

Intensive Margin - Calibration of Parameters

Assume the following isoelastic functional form for $\varphi(\cdot)$

$$\varphi(\ell_t) = \varphi_0 \frac{\varepsilon_\ell}{1+\varepsilon_\ell} (\ell_t)^{\frac{1+\varepsilon_\ell}{\varepsilon_\ell}} - \varphi_1 \quad \Rightarrow \quad \ell_t = \left(\frac{w_t \theta_t}{\varphi_0} \right)^{\varepsilon_\ell}.$$

This implies that the **elasticity of hours supply** with respect to the wage rate or the productivity parameter is

$$\frac{\partial \ln \ell_t}{\partial \ln w_t} = \frac{\partial \ln \ell_t}{\partial \ln \theta_t} = \varepsilon_\ell. \tag{1.1}$$

Calibrate:

- ε_ℓ for **strength of reaction**,
- φ_0 for level of ℓ_t
- φ_1 for level of $\varphi(\ell_t)$ (normalization, e.g. to zero).

Discuss: micro-elasticity ε_ℓ vs. macro-elasticity using `Blanchard_intensive`.

Extensive Margin - Endogenous Participation 1/3

The **participation decision** occurs before the hours decision and is in principle a **0/1-decision**. How to model this in this setting?

- **Problem**:
  - (a) if all households are the same either all choose 0 or 1 $\Rightarrow$ uninteresting,
  - (b) if households are different (e.g. they receive different shocks to the value of home production) how to keep track of them?
- **Answer**: We **convexify** the problem (e.g. see Hansen (1985) on indivisible labor)
  - **Possibility (a)**: Assume that households receive many, many home production shocks per period $\Rightarrow$ we can interpret the probability of participating as the time a household spends in participation.
  - **Possibility (b)**: Assume that households insure themselves against home production shocks (e.g. they pool income $\Rightarrow$ ex-ante expected per-period income is equal to ex-post per-period income).

Extensive Margin - Endogenous Participation 2/3

We use option (a): Households receive many shocks to home production

$$\text{home production} = \omega + h, \quad h \sim F(\cdot)$$

Define **household income** as $y_{v,t} = \delta_{v,t} w_t \ell_{v,t} \theta_t$. Households **participate if**

$$w_t \ell_{v,t} \theta_t - \varphi(\ell_{v,t}) > \omega + h_{v,t} \quad \Rightarrow \quad w_t \ell_{v,t} \theta_t - \varphi(\ell_{v,t}) - \omega = \underline{h}_{v,t}.$$

Given our assumption the **participation rate** $\delta_{v,t}$ (i.e. time spent in participation) is

$$\delta_{v,t} = F(\underline{h}_{v,t}).$$

The **consumption bundle** is defined as

$$Q_{v,t} = C_{v,t} - \delta_{v,t}\varphi(\ell_{v,t}) + (1 - \delta_{v,t}) \left[h^e_{v,t} + \omega \right] \tag{1.2}$$

with $h^e_{v,t} = F(\underline{h}_{v,t})^{-1} \int_{-\infty}^{\underline{h}_{v,t}} h \, dF(h)$.

Extensive Margin - Endogenous Participation 3/3

Observe that the **optimality** for hours is **unaffected** (use the Envelope Theorem)

$$\ell_{v,t}: \quad u'(Q_{v,t})\delta_{v,t}\varphi'(\ell_{v,t}) = \beta R_{t+1}\lambda_{v,t+1}\delta_{v,t}w_t\theta_t \Rightarrow$$
$$\varphi'(\ell_{v,t}) = w_t\theta_t \quad \Rightarrow \quad \ell_{v,t} = \ell_t.$$

Important: if $\ell_{v,t} = \ell_t \quad \Rightarrow \quad \underline{h}_{v,t} = \underline{h}_t \quad \Rightarrow \quad \delta_{v,t} = \delta_t$

Aggregate labor supply is $L_t^S = \delta_t \ell_t \theta_t N_t$.

The **extensive margin** is implemented in `Blanchard_extensive`.

Extensive Margin - Calibration of Parameters

Assume that $F(\cdot)$ follows a **Pareto distribution** with scale parameter k and shape parameter κ then the **non-participation rate** is

$$(1 - \delta) = \underline{h}^{-\kappa} k^{\kappa}$$

Calibrate:

- κ for **strength of reaction**,
- k for level of δ_t
- There is an explicit solution for the conditional expectation h_t^e

Discuss: How do changes in ω and θ affect both labor supply margins using `Blanchard_intensive`?

Outline

Assumptions

We add **government instruments** to a simple Blanchard model with the following **assumptions**

- Single country, small open economy.
- Firms face hd1 capital-adjustment costs.
- Labor productivity **grows exogenously** by rate g ($G = 1 + g$), model is **detrended**.
- Isoelastic felicity.
- Endogenous labor supply along intensive and extensive margin.
- A mass of households each facing a constant probability of death γ.
- Households of same age insure themselves against the risk of longevity.
- 3 markets: homogenous good (numéraire), labor, and assets.
- Government **issues debt** (perfect substitute for investing in firm or abroad)

Government instruments

We model the following **fiscal instruments** of the government

- Income taxes from workers: $\tau^W \in [0, 1]$
- Payroll taxes from firms: $\tau^F \geq 0$
- Lump-sum taxes/transfers from/to households: $\tau^I \in \mathbb{R}$
- Profit taxes from firms: $\tau^{prof} \in [0, 1]$
- Profit tax deductibility options for capital maintenance costs: $\phi_0^\tau \in [0, 1]$
- Consumption taxes: $\tau^C \geq 0$
- Unproductive (non-microfounded) government consumption: $C^G \geq 0$
- Benefits for non-participating households: $b \geq 0$ (instead of home production parameter ω)

Household problem

Adding our instruments changes the household problem to

$$\begin{aligned} V(A_{v,t}) &= \max_{C_{v,t},\ell_{v,t}} \ {}^{1}\!/\!_{\rho}\,(Q_{v,t})^{\rho} + \beta\gamma_{t+1}G^{\rho}V(A_{v,t+1}), \quad \text{s.t.} \\ \gamma_{t+1}GA_{v,t+1} &= R_{t+1}\left[A_{v,t} + y_{v,t} - pc_t C_{v,t}\right] \\ Q_{v,t} &= C_{v,t} - \delta_{v,t}\varphi(\ell_{v,t}) + (1-\delta_{v,t})h^{e}_{v,t} \\ y_{v,t} &= \delta_{v,t}(1-\tau^{W}_{t})w_t\ell_{v,t}\theta_t + (1-\delta_{v,t})b_t - \tau^{I}_{t} \end{aligned}$$

where the price of consumption is $pc_t = 1 + \tau^{C}_{t}$. The **behavior describing** equations are

$$\begin{aligned} \delta_{v,t}: &\quad \left[(1-\tau^{W}_{t})w_t\ell_{v,t}\theta_t - b_t\right]/pc_t - \varphi(\ell_{v,t}) = \underline{h}_{v,t} \quad \Rightarrow \quad \delta_{v,t} = \delta_t \\ \ell_{v,t}: &\quad \varphi'(\ell_{v,t})\cdot pc_t = (1-\tau^{W}_{t})w_t\theta_t \quad \Rightarrow \quad \ell_{v,t} = \ell_t \\ C_{v,t}: &\quad GQ_{v,t+1} = \left(\beta R_{t+1}\frac{pc_t}{pc_{t+1}}\right)^{\sigma} Q_{v,t} \end{aligned}$$

Firm problem

Maximize **firm value** (present value profits V):

$$V(K_t) = \max_{I_t, L_t^D} \chi_t + \frac{GV(K_{t+1})}{R_{t+1}}, \quad \text{s.t.}$$

$$\chi_t = Y_t - (1+\tau_t^F)w_t L_t^D - I_t - J_t - T^F \qquad \text{(dividend)}$$

$$T^F = \tau_t^{prof}\left[Y_t - (1+\tau_t^F)w_t L_t^D - \phi_0^\tau\left(J_t + \delta^K K_t\right)\right] \qquad \text{(profit tax)}$$

$$Y_t = f(K_t, L_t^D) \qquad \text{(hd1 production function)}$$

$$GK_{t+1} = (1-\delta^K)K_t + I_t \qquad \text{(capital law of motion)}$$

$$J_t = \psi/2 \cdot K_t(I_t/K_t - \delta^K)^2 \qquad \text{(capital adjustment costs)}$$

In steady state (i.e. $J = J_I = J_K = 0$) the **user-cost of capital** is

$$Y_K = \frac{r + \delta^K(1 - \phi_0^\tau \tau^{prof})}{1 - \tau^{prof}}.$$

The Government Sector

Per-period **revenue** and **expenditure** of the government are

$$Rev_t = T_t^F + \left(\tau_t^F L_t^D + \tau_t^W L_t^S\right) w_t + \tau_t^I N_t + \tau_t^C C_t,$$

$$Exp_t = C_t^G + (1 - \delta_t) b_t N_t.$$

Government debt evolves according to

$$GD_{t+1}^G = R_{t+1} \left(D_t^G - PB_t\right).$$

where **primary balance** is $PB_t = Rev_t - Exp_t$.

With perfect forward looking agents we have to set a **debt rule** such that detrended debt D^G converges to a number **bounded from above and below**.

Temporary Equilibrium

In period t we know

- all parameters and taxes (except for one)
- **predetermined** variables: K_t, D_t^F, D_t^G.
- guesses for **forward looking** variables: H_{t+1}, V_{t+1} and Ω_{t+1}.

Three markets have to clear: Labor ($L_t^D = L_t^S$), assets ($A_t = V_t + D_t^F + D_t^G$) and goods ($Y_t = C_t + I_t + J_t + C_t^G + TB_t$).

Government budget has to hold: $GD_{t+1}^G = R_{t+1}\left(D_t^G - PB_t\right)$ by changing at least one government instrument endogenously.

Respective code is `Blanchard_government`.

Outline

Implementation on the PC

Foresight variables: H, V and Ω, **Predetermined**: K and D^F

- We discuss code implementations: `Blanchard_government`.
- We look at the following
  1. budget balance switch
  2. calibration
  3. a simple reform with different budget closures
  4. changing exogenous debt path

  and interpret the results.

Home exercises: Ex. 8 and 9.