# General Equilbrium Policy Analysis

# Lecture 1

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at the Institute for Advanced Studies, Vienna

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# Outline

# Type of questions to be answered

- *"What are the medium-run effects on GDP of revenue-neutrally shifting emphasize from the income to the value added tax in Spain?"*
- *"How much would the effective retirement age in Austria have to change over time to offset the effects from increasing population aging keeping all other pension system parameters constant?"*
- *"What are the consequences of increasing yearly migration flows to the United Kingdom to its public budgets?"*

# Goals of the course

The **goals of the course** are...

- to introduce students to tools to **answer policy questions** such as those from the previous slide,
- to give a thorough **understanding of how these tools work** (look inside the black back instead of using front end modeling tools),
- and to do this **step by step** starting from the easiest model by adding model components sequentially,
- while giving a detailed **documentation** of the derivations and the **computer codes**.

# Scope of the course

Focus on **dynamic computable general equilibrium** models mainly for **fiscal policy** simulation, in particular **transitional effects** of policies related to the **tax, social security and the pension system** with an emphasis on the household sector.

What is **not covered**:

- Aggregate shocks (DSGE).
- Certain idiosyncratic shocks (heterogenous agents literature).
- Inflation and monetary policy.
- Normative analyses.
- A detailed modeling of the production side (e.g. various sectors of the economy, often done as a static CGE exercise).

# Tentative outline of the course

- **1. Lecture**: Introduction to solution algorithm, compute transitions for simple Ramsey model
- **2. Lecture**: Introduction to OLG, Blanchard model in open economy with demographic change
- **3. Lecture**: Endogenous labor supply, introduction of government and simulation of fiscal policy
- **4. Lecture**: Unanticipated shocks, Walras' Law, effective taxation and progressivity and some principle comments on debugging and calibration
- **5. Lecture**: Introduction to Gertler model and probabilistic aging.
- **6. Lecture**: Introduction to Auerbach-Kotlikoff model, earnings-related pensions.
- **7. Lecture**: Intergenerational distribution, calibration and possible extensions to the Auerbach-Kotlikoff model.

**Material** from `https://sites.google.com/site/schusterphilip/`: Manuscript (under continuous development), Slides (available before the lecture), MATLAB codes.

# Time slots

- **1. Lecture**: April 13, 2014, 16:30 - 18:30, Room IT A014
- *1. Practice Session*: April 15, 2014, 11:00 - 13:00, Room IT A014
- **2. Lecture**: April 15, 2014, 16:30 - 18:30, Room IT A014
- *2. Practice Session*: April 20, 2014, 10:30 - 12:30, Room IT A014
- **3. Lecture**: April 20, 2014, 16:30 - 18:30, Room IT A014
- **4. Lecture**: April 23, 2014, 16:30 - 18:30, Room IT A014
- *3. Practice Session*: April 27, 2014, 11:00 - 13:00, Room IT A014
- **5. Lecture**: April 27, 2014, 16:30 - 18:30, Room IT A014
- **6. Lecture**: April 30, 2014, 16:30 - 18:30, Room IT A014
- *4. Practice Session*: May 4, 2014, 11:00 - 13:00, Room IT A014
- **7. Lecture**: May 4, 2014, 16:30 - 18:30, Room IT A014

# Homework

- **1. Homework**: assigned in 1. lecture, due before 2. lecture
- **2. Homework**: assigned in 2. lecture, due before 3. lecture
- **3. Homework**: assigned in 3. lecture, due before 5. lecture
- **4. Homework**: assigned in 5. lecture, due before 7. lecture
- **Final Project**: assigned in 7. lecture, due by June 7.

# Grading

**Grading** contributes 1/3 to the overall grade ('*Applied Theory*') and is based on **three components**

- Active participation in class (20%)
- Home exercises (in groups of 2-3 persons) (30%)
- Final project (in groups of 2-3 persons) (50%)

**Tentative final project**: A thorough calibration of the presented Auerbach-Kotlikoff model to an EU country of your choice. Simulation and interpretation of some defined reforms.

# Outline

# Solving Deterministic Dynamic Perfect Foresight Models

**Principle challenge is the computation of the transition**:

- An agent's decision depends on her own future decisions as well as on **future decisions of other agents** (through prices!)
- Agents have to make **time-consistent** optimal decisions, i.e. ex-post they must not want to alter their choices (as they would with adaptive expectations, i.e. expectations based on the past)

**Solution** is a **path of prices** for all $n$ markets under the condition that agents' **expectations are fully consistent** with the realizations.

**Two options**:

- **One shot approach**: Stack all equilibrium conditions of all periods together (including the time-consistency conditions) and solve for a path of prices for all markets in one go (using a standard numerical root finding algorithm)
- **Split the problem in subproblems** which is a more robust approach especially for large scale problems.

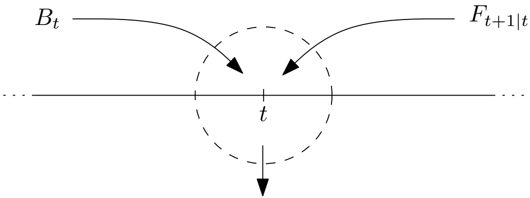
# Fair Taylor algorithm 1/3

The **key for option 2** is to **separate** the tasks of finding market clearing prices for all periods and establishing time-consistency of agents' expectations. A prerequisite is that the **characterization of the solution** can be formulated in a **recursive way** (i.e. a system of first-order difference equations).

- We differentiate between:
  - A set of prices $p$
  - A set of predetermined, **backward looking** variables $B$, like the capital stock.
  - A set of **forward looking** variables $F$, like future wages or profits.
  - A set of **parameters** $Z$, like utility parameters, tax rates, etc.
- Define a **temporary equilibrium** in $t$ as a price vector $p_t$ which for a given guess about the foresight variables $F_{t+1|t}$ sets all excess demands to zero, i.e. $\zeta_n(p_t, B_t, F_{t+1|t}, Z_t) = 0, \ \forall n$.

# Fair Taylor algorithm 2/3

Figure: The subproblem at time $t$




$$p_t \text{ such that } \zeta_n(p_t, B_t, F_{t+1|t}, Z_t) = 0, \ \forall n$$

$$B_{t+1} = \Pi(p_t, B_t, F_{t+1|t}, Z_t)$$

$$F_{t|t} = \Gamma(p_t, B_t, F_{t+1|t}, Z_t)$$

Solving for all temporary equilibria from $t = 1$ to $t = T - 1$ leaves us with an updated guess matrix, $\Lambda(F^i)$. In the **Fair-Taylor algorithm** the guess is updated according to

$$F^{i+1} = \psi \Lambda \left(F^i\right) + (1 - \psi) F^i, \qquad \psi \in (0, 1]$$

until convergence, i.e. $d(F^{i+1}, F^i) < \epsilon$.

# Fair Taylor algorithm 3/3

The **steps of a full program** for computing a transition path:

1. load model **parameters**
2. **calibrate** the model
3. compute the **Jacobian** of the system $\Lambda(\cdot)$ (only necessary if generalized version of Fair-Taylor algorithm is used)
4. compute the **initial steady state** at $t = 1$
5. enter a **shock** to system, e.g. a policy reform
6. compute the **final steady state** at $t = T$
7. solve for the **transition path**
   a) make a **guess** for $F$
   b) compute a **path of temporary equilibria** starting from $t = 1$ to $t = T - 1$
   c) **update** the guess for $F$
   d) repeat steps b) to c) until **convergence**

# Outline

# Assumptions

To illustrate the solution algorithm we choose a simple Ramsey model with the following **assumptions**

- Single country, closed economy.
- Log felicity.
- Exogenous labor supply.
- Single representative firm owned by infinitely-lived representative household.
- 3 markets: homogenous good (numéraire), labor, and assets.

# Household problem 1/2

$$U(A_t) = \max_{C_t} \; u(C_t) + \beta U(A_{t+1}), \quad \text{s.t.}$$
$$A_{t+1} = R_{t+1}\left[A_t + w_t L_0 - C_t\right]$$

where $R_{t+1} = 1 + r_{t+1}$. Note **timing convention**: Decisions and flows at the **beginning of the period**. Define the shadow price of assets as $\lambda_t = U'(A_t)$ then the **optimality** and **envelope** conditions are:

$$C_t : \quad u'(C_t) = \beta\lambda_{t+1}R_{t+1}$$
$$A_t : \quad \lambda_t = \beta\lambda_{t+1}R_{t+1}$$

Hence, the **Euler equation** (for log felicity) is

$$u'(C_t) = \beta R_{t+1}u'(C_{t+1}) \quad \Rightarrow \quad C_{t+1} = \beta R_{t+1}C_t$$

# Household problem 2/2

Recursively insert to rewrite the **budget constraint** as

$$A_t = C_t - w_t L_0 + \frac{1}{R_{t+1}} \Big[ C_{t+1} - w_{t+1} L_0 + \frac{1}{R_{t+2}} \Big[ C_{t+2} - w_{t+2} L_0 + \dots$$

$$\Leftrightarrow \mathcal{W}_t = A_t + H_t \quad \text{where}$$

$$\mathcal{W}_t = C_t + \frac{1}{R_{t+1}} \Big[ C_{t+1} + \frac{1}{R_{t+2}} \Big[ C_{t+2} + \dots$$

$$H_t = w_t L_0 + \frac{1}{R_{t+1}} \Big[ w_{t+1} L_0 + \frac{1}{R_{t+2}} \Big[ w_{t+2} L_0 + \dots = w_t L_0 + \frac{H_{t+1}}{R_{t+1}}$$

Insert the **Euler equation** in the definition of $\mathcal{W}_t$

$$\mathcal{W}_t = C_t + \frac{1}{R_{t+1}} \Big[ C_t \beta R_{t+1} + \frac{1}{R_{t+2}} \Big[ C_t \beta^2 R_{t+1} R_{t+2} + \dots$$

$$\mathcal{W}_t = C_t \cdot \sum_{s=t}^{\infty} \beta^{s-t} = C_t \cdot \Omega_t = C_t \cdot \frac{1}{1-\beta}.$$

Hence, the **consumption function** is

$$C_t = \Omega_t^{-1} (A_t + H_t).$$

# Firm problem 1/2

Maximize **firm value** (present value profits $V$):

$$V(K_t) = \max_{I_t, L_t^D} \chi_t + \frac{V(K_{t+1})}{R_{t+1}}, \quad \text{s.t.}$$

$$\chi_t = Y_t - w_t L_t^D - I_t \qquad \text{(per-period profits = dividend)}$$

$$Y_t = f(K_t, L_t^D) \qquad \text{(hd1 production function)}$$

$$K_{t+1} = (1 - \delta^K) K_t + I_t \qquad \text{(capital law of motion)}$$

Define the shadow price of capital as $q_t = V'(K_t)$ then the two **optimality** and the **envelope** conditions are:

$$I_t : \quad q_{t+1} = R_{t+1}$$

$$L_t^D : \quad Y_{L_t^D} = w_t$$

$$K_t : \quad q_t = Y_{K_t} + \frac{q_{t+1}}{R_{t+1}}(1 - \delta^K)$$

# Firm problem 2/2

## Theorem

*Hayashi's theorem. Firm value and capital stock are related through Tobin's-q*

$$q_t K_t = V_t, \quad \forall t$$

## Proof

*See manuscript.*

Combine Hayashi's theorem with the law of motion and the optimality condition for investment to get a **relationship of investments and future profits**

$$I_t = \frac{V_{t+1}}{R_{t+1}} - (1 - \delta^K)K_t$$

# Temporary Equilibrium

In period $t$ we know

- all parameters
- **predetermined** variable: $K_t$.
- guesses for **forward looking** variables: $H_{t+1}$ and $V_{t+1}$.

**Three markets** have to clear: Labor ($L_t^D = L_0$), assets ($A_t = V_t$) and goods ($Y_t = C_t + I_t$).

1. $L_t^D = L_0 \quad \Rightarrow \quad w_t$
2. $I_t = \frac{V_{t+1}}{R_{t+1}} - (1 - \delta^K)K_t \quad \Rightarrow \quad V_t, K_{t+1}$
3. $H_t = w_t L_0 + \frac{H_{t+1}}{R_{t+1}}$ and $A_t = V_t \quad \Rightarrow \quad C_t$
4. $Y_t = C_t + I_t \quad \Rightarrow \quad R_{t+1}$

$R_{t+1}$ is found numerically (`Ramsey`) or using some algebra analytically (`Ramsey_simple`).

# Outline

# Implementation on the PC

Some principle **notation**: A calibration value of variable $X$ is scalar `X0`. The time vector of $X$ from $t = 1$ to $t = T$ is a row vector `X`.

**Foresight variables**: $H$ and $V$, **Predetermined**: $K$

- We discuss code implementations: `Ramsey` and `Ramsey_simple`.
- We walk through all functions step by step.
- We look at 2 simple reforms:
  1. a negative shock to the capital stock and
  2. an exogenous increase in labor supply

  and interpret the results.

**Home exercises due before lecture 2**: Ex. 2 and 3.