

Labor Market Policy Instruments and the Role of Economic Turbulence

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Abstract

This paper uses a dynamic model of search unemployment and bilateral wage bargaining to characterize optimal labor market policy in a possibly turbulent environment. A firing externality, generated by the existence of a partial unemployment insurance system, distorts the pre-policy equilibrium along three margins: job creation, job acceptance, and job destruction. Optimal policy is characterized by a wage tax, a firing tax, and a hiring subsidy. Endogenous job acceptance demands that a firing tax and a hiring subsidy have to be set equal in any case and cannot be used to correct for the possible failure of the Hosios condition. In that case the optimal policy mix has to be extended by either an output or recruitment tax/subsidy. It is further shown that the derived policy mix is robust to the introduction of economic turbulence in form of state-dependent worker transitions between skill classes. This is crucial as intergroup cross-subsidization schemes, like in-work benefits targeted at low-skilled workers, are rendered considerably less effective in that case. Instead of cross-subsidization from high- to low-skilled workers or from firing firms to unemployed workers, the paper identifies a scheme involving redistribution from firing to hiring firms to be optimal.

Keywords: search and matching, employment subsidies, economic turbulence, policy spill-over

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1 Introduction

Times of high unemployment always inspire debates on the role of labor market policy. While most scholars agree that unemployment rates in the OECD, especially of low-skilled workers, are excessively high, there is little consensus on what should be done about it, leading to a variety of policy advice. Take for example the recent debate in Germany on the controversial subject of which of two specific policy instruments should be introduced: wage subsidies (e.g. Sinn et al., 2006) versus hiring subsidies (e.g. Brown et al., 2007). While there exists much empirical work focusing on the relative effectiveness of specific instruments, the aim of this paper is to develop a theoretical characterization of optimal policy. I will employ a dynamic Diamond-Mortensen-Pissarides (DMP) model, rich enough to incorporate three important decision margins: job creation, job acceptance, and job destruction, in a potentially turbulent environment. Turbulence is introduced to the model, in the spirit of Ljungqvist and Sargent (1998), as state-dependent transitions of workers between skill classes, i.e. unemployed workers lose their skills in the course of time. This adds an additional channel for policy spill-over to the framework which plays an important role in case of policy targeting. Possible inefficiencies of the initial, pre-policy equilibrium come in the form of a typical search externality and a firing externality, which I will focus on in more detail. The need to finance an existing partial unemployment insurance (UI) system, not internalized by firms, gives rise to the latter externality. I introduce a set of policy instruments: wage, output and firing taxes as well as wage, hiring and recruitment subsidies, while I do not allow for taxation of unemployed workers.

The analysis shows that the optimal policy mix can be characterized by a wage tax as a fiscal instrument set to finance the UI system and a 'firing tax equal hiring subsidy'-scheme similar to Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), representing redistribution from firing to hiring firms, to correct for the distortions¹. This contrasts the results of Blanchard and Tirole (2008) (henceforth BT08) who, in a static approach, neglecting job creation and acceptance, characterize optimal policy by redistribution from firing firms to unemployed workers, which resembles the experience rating system implemented in the United States². The derived optimal policy mix has to be expanded by either an output or a recruitment tax/subsidy in case of 'unbalanced' search externalities to correct for the failure of the Hosios (1990)-condition. An important finding is that the characterization of optimal policy is robust to the introduction of economic turbulence. This is crucial as a lot of existing policy advice is rendered considerably less effective in

¹Ricci and Waldmann (2011) derive a similar policy recommendation to correct for a hold-up problem caused by contractual incompleteness concerning firm-specific training. In contrast, in my paper workers' skills are general and not attached to specific firms.

²The fundamental insights were first described by Feldstein (1976) and Topel (1983).

that case. The cross-financed wage subsidy scheme for low-skilled workers, representing redistribution from high- to low-skilled workers, as proposed by Mortensen and Pissarides (2003) (henceforth MP03) is an example for such a policy advice. They assume skill classes to operate in complete juxtaposition, except for the connection via the government's budget constraint, which underestimates the adverse effect on high-skilled workers who become pickier concerning their acceptance and continuation decision as their fall back option increases with the wage subsidy for low-skilled workers. In addition the skill composition deteriorates as a result of such a targeting scheme.

This paper relates to several strands of the literature. As mentioned above there are numerous empirical studies³ that deal with estimating the employment effects of various policy instruments. But typical inference from comparing treated and untreated individuals to evaluate 'big scale' policies faces the following problems. First, a policy has actually to be in place. Second, 'big scale' policies will induce general equilibrium effects which lead to a violation of the necessary 'stable unit treatment value assumption' (see Angrist et al., 1996). In their study on counseling, Cahuc and Barbanchon (2010) argue how micro evaluations neglecting crowding out, adverse spill-over effects on non-targeted persons, and other equilibrium effects can lead to misguided policy advice. I therefore base my analysis on a model of equilibrium search unemployment rich enough to capture those effects. Dynamic DMP models have been widely used to evaluate different labor market policy instruments ever since the influential paper of Mortensen and Pissarides (1994). However, the conclusions so far are mixed. While Bovenberg et al. (2000) and Cardullo and van der Linden (2006) argue that wage subsidies can substantially reduce unemployment, Boone and van Ours (2004) and Oskamp and Snower (2008) find no such effect. What these and similar studies typically have in common is that they embed the frictional labor market in complex CGE-models which makes it hard to disentangle different effects or draw conclusions concerning the optimal design of policy which is at the heart of my paper. A more theoretical treatment - probably most closely related to this paper - is provided by MP03. They analytically derive optimal policy before presenting some simulation results for non-optimal policy schemes. I will extend their analysis on several dimensions. First, in their optimal policy characterization, MP03 solely concentrate on the distorting effects of subsidies and taxes while fiscal effects are suppressed by allowing for non-distortionary consumption taxes. Hence, the firing externality in the spirit of BT08 does not play a role in their setting. Second, I introduce an additional margin, namely job acceptance, which will alter optimal policy if search externalities are 'unbalanced'. Third

³Empirical evidence on the effects of wage subsidies is summarized by Katz (1996) for the United States, Bell et al. (1999) for the United Kingdom, and Bonin et al. (2002) for Germany. Boockmann et al. (2007) provide some evidence on 'hiring subsidy'-like grants in Germany. A review on the empirical effects of employment protection is presented in Lazear (1990) and Skedinger (2011).

and most importantly, I introduce economic turbulence and discuss its role for policy design. Another closely related paper is Michau (2011) who extends the BT08-setting to a dynamic DMP framework. He explicitly models the UI problem with risk-averse workers and finds that the welfare maximizing allocation is characterized by full insurance and output maximization. As Nash bargained wages with positive bargaining power of the workers are incompatible with full insurance, he finds that in a second best a social planner would reduce labor market tightness, implemented by a positive spread between a firing tax and a hiring subsidy. This is done to reduce wages and therefore decrease under-provision of UI. I do not consider this trade-off between insurance and output maximization here by conditioning my analysis on the existence of a partial UI system as the focus lies on role of economic turbulence in designing policy. The idea of economic turbulence is inspired by work of Ljungqvist and Sargent (1998). While this strand of the literature, including Pissarides (1992), Ljungqvist and Sargent (2004), and Den Haan et al. (2005), is concerned with the influence of skill depreciation during unemployment on the persistence of unemployment, its implication for policy design has received little attention so far.

The outline of the paper is as follows. First, a simple intragroup model is developed featuring only one skill class. The optimal policy mix, implementing the social planner's solution, is characterized, which will provide good intuition for the more complex intergroup model discussed in section 3. I extend the intragroup by an additional skill class and allow for redistribution as well as economic turbulence in form of state-dependent transitions of workers between the skill classes. After showing how the characterization of optimal policy in the intergroup model relates to the intragroup case I present some simulation results in order to highlight also the quantitative dimension of my findings.

2 A simple intragroup model

The model is based on the standard dynamic Mortensen and Pissarides (1994)-framework enriched by endogenous acceptance. In this section I consider only one skill class. There are two types of rational, forward looking agents: workers and firms. Labor force L is comprised of atomistic risk-neutral workers. The assumption of risk-neutrality is discussed more thoroughly in section 2.2. There is a sufficiently large number of risk-neutral firms that can enter the labor market instantaneously but are subject to per-period net flow costs c for posting a vacancy. For production each firm needs one worker who will inelastically supply one unit of labor if employed. The three decision margins: job creation (θ), job acceptance (\underline{x}), and job destruction (\hat{x}) are best understood when looking at the life cycle of a job. First, firms decide to post vacancies according to a free entry condition which fixes labor market tightness θ . The search friction implies that it takes time to fill a

vacancy during which a firm has to pay per-period gross posting costs C that are reduced by a recruitment subsidy R to $c = C - R$. Eventually a worker and a firm are matched according to a matching technology m . This can be interpreted as meeting for a job interview. Only then the agents will learn how well suited an applicant is for the specific job. This is modeled as drawing a job-specific productivity x from a known distribution $G(\cdot)$. If the realization of the draw is higher than the according reservation productivity, referred to as 'outside' cut-off, i.e. $x > \underline{x}$, the job is started and the firm receives a one-time hiring subsidy H . Technically, this is one of the main difference compared to MP03 who assume that every job is created at maximum idiosyncratic productivity, trivializing the acceptance decision because job offers are rejected with probability zero.⁴ During production the firm receives net off tax output $(1 - \tau)x$ and an in-work benefit or wage subsidy D that partly compensates for the wage w , stemming from a Nash bargaining game, it has to pay to the worker. A new idiosyncratic productivity shock arrives with probability π^n . If a new draw is lower than the endogenous 'inside' cut-off, i.e. $x < \hat{x}$, the job is destroyed and the firm has to pay a separation or firing tax F . To summarize the featured instruments. Three different subsidies will be analyzed: a periodic lump-sum wage subsidy (D), a one-time hiring subsidy to the firm (H), and a recruitment subsidy (R). On the other hand I will analyze three distortionary taxes, namely: firing taxes (F), linear output taxes (τ), and linear⁵ wage taxes (t). An important assumption I make is that unemployed workers cannot be taxed. Part of the value of non-work is home production which cannot be transformed into tax revenue. This rules out non-distortionary consumption taxation. Analytically, the model can be described as follows.⁶

As usual for this kind of framework an aggregate matching function $\mathcal{M}(u, v)$, which maps the stock of unemployed (u) and the stock of vacancies (v) into the flow of new matches (\mathcal{M}), is assumed to be homogeneous of degree one with elasticity w.r.t. u of $0 < \eta < 1$. Defining labor market tightness as $\theta \equiv \frac{v}{u}$ results in the matching probability functions (2.1) and (2.2) for firms and workers,

$$\text{prob. of a match for the firm: } \frac{\mathcal{M}(u, v)}{v} = q(\theta), \quad (2.1)$$

⁴In an alternative interpretation this relates to Hall (2005) who also allows for less qualified persons to apply. In contrast to my analysis, he assumes that the qualification of an applicant is not completely revealed to the employer in the first meeting. This can only be resolved if the employer decides to costly evaluate the application.

⁵The linearity assumption does not drive the fundamental results but helps to keep the mathematics straightforward. Note that the lump-sum component of the wage subsidy going to the worker and the linear component t can mimic a regressive or progressive tax schedule. The implementation of the efficient allocation, as derived in section 2.2, does not require a wage subsidy.

⁶The notation is based on Pissarides (2000) with few exceptions. A description of all used variables can be found in appendix section G.

$$\text{prob. of a match for the worker: } \frac{\mathcal{M}(u, v)}{u} = \theta q(\theta), \quad (2.2)$$

with $q'(\cdot) < 0$, $q''(\cdot) < 0$ and $\mathcal{M}(u, v) \leq \min\{u, v\}$. Further define $q^f \equiv q(\theta)(1 - G(\underline{x}))$ and $q^w \equiv \theta q(\theta)(1 - G(\underline{x}))$ as the joint probabilities of matching and accepting. A worker can be either employed (e) or unemployed (u), that is I abstract from transitions into and out of labor force, hence $e + u = L$. Each state is associated with a specific present value, U for being unemployed and $W(x)$ or $\hat{W}(x)$ for becoming or being employed, respectively. A firm participating in the labor market can be in two states. Either it is looking for a worker which has value V or it is employing a worker which gives $J(x)$ or $\hat{J}(x)$. In general, the hat-notation always indicates that the worker or the firm have already been in the same state before the arrival of a shock. Or put differently, 'without hat' can be referred to as the initial or 'outside' value while 'with hat' denotes the continuation or 'inside' value. Given the assumption of perfect capital markets, where r denotes the exogenous interest rate, I can write both asset equations of working as follows

$$rW(x) = (1 - t)w(x) + \pi^n \left[(1 - G(\hat{x})) \hat{W}^{\hat{e}} + G(\hat{x})U - W(x) \right], \quad (2.3)$$

$$r\hat{W}(x) = (1 - t)\hat{w}(x) + \pi^n \left[(1 - G(\hat{x})) \hat{W}^{\hat{e}} + G(\hat{x})U - \hat{W}(x) \right]. \quad (2.4)$$

A just recently employed worker's felicity equals after-tax wage income $(1 - t)w(x)$ or $(1 - t)\hat{w}(x)$, respectively. When a shock arrives he loses $W(x)$ and gains U if the new productivity draw x is lower than the 'inside' cut-off \hat{x} , hence with probability $G(\hat{x})$. With probability $(1 - G(\hat{x}))$ he gets $\hat{W}^{\hat{e}}$, which denotes the conditional expectation⁷ of the value of being employed. The asset value of being unemployed is given by

$$rU = z + q^w (W^e - U), \quad (2.5)$$

where z denotes the value of non-work which is composed of unemployment compensation b and home production h in a linear way, $z \equiv b + h$. Turning to the firms' side the asset value of a vacancy can be written as

$$rV = -c + q^f (J^e + H - V), \quad \text{where } c = C - R. \quad (2.6)$$

Two subsidies enter this relationship. In case of an accepted match the firm has to give up the value of a vacancy V but gets the expected value of a job for the firm J^e plus a hiring subsidy H . The gross flow costs of maintaining a vacancy C minus the recruitment subsidy R give the net costs c . As free entry is imposed and V is decreasing in θ , in

⁷The conditional expectation of some random variable $X(x)$ w.r.t. \hat{x} is defined as $E(X(x)|x > \hat{x}) \equiv X^{\hat{e}} = \int_{\hat{x}}^{\infty} \frac{X(\tilde{x})}{1 - G(\hat{x})} dG(\tilde{x})$. Note the difference in notation compared to $E(X(x)|x > \underline{x}) \equiv X^e$.

equilibrium V is driven down to zero which will pin down θ , hence

$$V = 0 \Rightarrow \theta. \quad (2.7)$$

The asset values of a job are given in (2.8) and (2.9),

$$rJ(x) = (1 - \tau)x - w(x) + D + \pi^n \left[(1 - G(\hat{x}))\hat{J}^e - G(\hat{x})F - J(x) \right], \quad (2.8)$$

$$r\hat{J}(x) = (1 - \tau)x - \hat{w}(x) + D + \pi^n \left[(1 - G(\hat{x}))\hat{J}^e - G(\hat{x})F - \hat{J}(x) \right]. \quad (2.9)$$

In the current period a firm receives after-tax⁸ production $(1 - \tau)x$ minus wage rate $w(x)$ or $\hat{w}(x)$ plus a wage subsidy D .⁹ In case of a separation, which occurs with probability π^n and the probability of $x < \hat{x}$, a firm has to pay a firing tax F . Observe that given the wage determination explained below a firm and a worker will always mutually agree to destroy or create a job, i.e. both sides have the same reservation productivities. Hence, the notions of a 'firing' and a 'separation' tax are equivalent. The reservation productivities are pinned down by the following conditions

$$J(\underline{x}) + H = 0 \Rightarrow \underline{x}, \quad (2.10)$$

$$\hat{J}(\hat{x}) + F = 0 \Rightarrow \hat{x}. \quad (2.11)$$

The first relation states that after meeting for an interview and observing the match specific productivity x , a job will only be generated if the value of a job including the one-time hiring subsidy is non-negative. The second condition reflects that a firm will only want to continue a job if its value covers at least the firing tax. Wages are determined via Nash bargaining and are renegotiated every time a shock arrives. The Nash wages are given as solutions to the following optimization problems, where the weight ω can be interpreted as the worker's bargaining power,

$$w(x) = \operatorname{argmax} (W(x) - U)^\omega (J(x) + H)^{1-\omega}, \quad (2.12)$$

$$\hat{w}(x) = \operatorname{argmax} \left(\hat{W}(x) - U \right)^\omega \left(\hat{J}(x) + F \right)^{1-\omega}. \quad (2.13)$$

Lemma 2.1. *If $F = H$ then $w(x) = \hat{w}(x)$.*

⁸One might argue that an output tax is a rather abstract instrument in contrast to for example a cash-flow tax. Note however that cash-flow taxation of form $\tilde{\tau} [x - w(w)]$ would just imply a mixture of output taxation and subsidization of wage costs. Later is not directly implemented in the model, but can be mimicked by adjusting the employee's wage tax t because of Nash bargaining.

⁹Note that with Nash bargaining it does not matter economically whether the wage subsidy is given to the worker or the firm but the interpretation of w changes. In my setting w and $(1 - t)w$ are interpreted as gross and net wages received by the worker already including all subsidies.

Proof. Note that $w(x) = \hat{w}(x)$ implies $W(x) = \hat{W}(x)$ and $J(x) = \hat{J}(x)$ by construction. If $F = H$, the two problems (2.12) and (2.13) are identical. ■

The result of this lemma is more general and can be extended to non-linear utility and non-linear wage income taxation¹⁰. Given my assumptions the equilibrium 'outside' and 'inside' wage rates can be solved for explicitly¹¹

$$w(x) = (1 - \omega) \frac{z}{1 - t} + \omega((1 - \tau)x + D + c\theta + rH) - \omega\pi^n(F - H), \quad (2.14)$$

$$\hat{w}(x) = (1 - \omega) \frac{z}{1 - t} + \omega((1 - \tau)x + D + c\theta + rF). \quad (2.15)$$

Observe that the 'inside' and 'outside' wage distributions are directly related to the productivity distribution $G(\cdot)$ for x larger than the respective cut-off. A wage subsidy D will increase both wage schedules by the share the worker can claim in the process of bargaining ωD . While a recruitment subsidy R , which is included in c , decreases both wages to the same extent, they respond differently to a hiring subsidy H and a firing tax F . A hiring subsidy will increase the 'outside' wage of a worker while it does not affect the 'inside' wage as the subsidy is already sunk. A firing tax will abate 'outside' wages as firms are more cautious about hiring workers because they eventually have to pay F . In contrast, 'inside' wages will be inflated by F because firms are more willing to hold on to workers once they are employed. The relationship of 'outside' and 'inside' wage is simply $w(x) = \hat{w}(x) - (r + \pi^n)\omega(F - H)$. At last, in equilibrium the government's budget constraint has to hold,

$$\begin{aligned} 0 = & (L - u)\bar{w}t + (L - u)\bar{x}\tau + (L - u)\pi^n G(\hat{x})F \\ & - uq^w H - \theta u R - (L - u)D - ub. \end{aligned} \quad (2.16)$$

where \bar{w} and \bar{x} denote average wage and productivity, respectively. The first line represents tax income from the wage tax, the output tax and the firing tax. The second line gives expenditure on hiring, recruitment, and wage subsidies as well as unemployment benefits.

2.1 Equilibrium

The equilibrium vector $\langle u, \theta, \underline{x}, \hat{x} \rangle$ is pinned down by the four equations (2.17) to (2.20)¹². Equilibrium is partly recursive, i.e. only (2.17) and (2.18), henceforth referred to as the JD-JC system, have to be solved simultaneously for θ and \hat{x} after inserting (2.19). The job creation (JC) curve, which is derived from the free entry condition, equates expected

¹⁰Insert $u(w(x) - T(x))$ in (2.3) and $u(\hat{w}(x) - T(x))$ in (2.4), with the mild conditions $u'(\cdot) > 0$ and $w(x) - T(x) > 0$ for otherwise arbitrary functions $u(\cdot)$ and $T(\cdot)$. The proof still holds.

¹¹See appendix section C for the derivation.

¹²See appendix section C for a detailed derivation of (2.17) to (2.20).

gain and cost of a vacancy

$$\text{JC} : (1 - \omega) \left(\frac{(x^e - \hat{x})(1 - \tau)}{\pi^n + r} - F + H \right) - \frac{c}{q^f} = 0. \quad (2.17)$$

The first term is the expected gain of job creation for a firm, i.e. the firm's after-tax share of excess output discounted by $\pi^n + r$. The gain is additionally raised or lowered depending on whether the hiring subsidy H exceeds the firing tax F , or vice versa. The second term reflects the expected costs of job creation, i.e. the net flow cost c times the average duration of a vacancy $1/q^f$.

$$\begin{aligned} \text{JD} : (1 - \tau)\hat{x} + D + \frac{\pi^n(1 - \tau)}{\pi^n + r} \int_{\hat{x}}^{\infty} (\tilde{x} - \hat{x}) dG(\tilde{x}) \\ - \frac{z}{1 - t} + rF - \frac{\omega}{1 - \omega} c\theta = 0. \end{aligned} \quad (2.18)$$

The first line of the job destruction (JD) condition, which represents the 'inside' cut-off condition, gives the lowest acceptable joint inside value of a job, i.e. the after-tax reservation product plus a wage subsidy D and the option value of keeping a worker as her productivity might change. The second line can be interpreted as the joint outside value, which increases in z and θ , as both raise the worker's outside option, and decreases in F . The analytic relationship of the 'outside' to the 'inside' productivity cut-off is novel compared to other studies that do not take endogenous job acceptance into account, i.e.

$$\underline{x} = \hat{x} + \frac{(\pi^n + r)}{(1 - \tau)}(F - H). \quad (2.19)$$

Observe that both cut-offs coincide in a policy free environment where $F = H = 0$. A hiring subsidy H will put a wedge between those cut-offs in a way that agents more easily accept than destroy a job ($\underline{x} < \hat{x}$). A firing tax F has the opposite consequence, $\underline{x} > \hat{x}$. Having derived all three decision variables θ , \hat{x} , and \underline{x} , I can compute unemployment u . Just insert in the typical Beveridge curve (2.20), which is derived by setting the change in u , i.e. $\dot{u} = (L - u)\pi^n G(\hat{x}) - uq^w$, to zero, i.e.

$$u = \frac{\pi^n G(\hat{x})}{\pi^n G(\hat{x}) + q^w} \cdot L. \quad (2.20)$$

As mentioned, the recursion of the system reduces the problem to solving only two equations simultaneously. Therefore, I can conveniently analyze comparative statics in the JD-JC diagram¹³, drawn in the θ - \hat{x} -space (see Pissarides, 2000). The JC-curve is sloping downward because firms post fewer vacancies the higher \hat{x} , as average duration of a job

¹³See appendix section F.1 for more details.

decreases in \hat{x} . The JD-curve slopes upward because workers want to terminate jobs more easily the higher θ , as their outside options increase in labor market tightness. Hence, the curves intersect at most once, as illustrated by figure 2.1, which makes the equilibrium unique in case of existence. I will now shortly address the effects of uncompensated changes in my policy instruments¹⁴. A wage subsidy D has no effect on the JC-curve but shifts out the JD-curve. Hence, equilibrium labor market tightness θ will go up, the reservation productivities $\underline{x} = \hat{x}$ will fall, leading to more job creation, more acceptance and less destruction. Therefore, unemployment will unambiguously decrease. A hiring subsidy H works quite differently. While there is no effect on the JD-curve, the JC-curve will shift outward. This raises labor market tightness and consequently job creation as well as job destruction. Relative to job destruction, job acceptance is boosted, i.e. $\underline{x} < \hat{x}$. Whether job acceptance rises or falls in absolute terms is ambiguous. Proposition 2.1 states a condition for the direction of the absolute effect.

Proposition 2.1. *Hiring subsidy and job acceptance:* *A hiring subsidy can lead to more or less job acceptance. Assume for simplicity that $t = \tau = 0$. Whenever $\nabla^{-1}\omega c(1 - \Psi) < \pi^n + r$ the effect of H on \underline{x} will be negative, leading to more job acceptance.*

Proof. Differentiating (2.19) w.r.t. H gives $\frac{\partial \underline{x}}{\partial H} = \frac{\partial \hat{x}}{\partial H} - (\pi^n + r)$. Inserting for $\frac{\partial \hat{x}}{\partial H}$ derived using the implicit function theorem and rearranging completes the proof.¹⁵ ■

A firing tax F has very similar but inverted effects compared to a hiring subsidy H . While the JC-curve moves inward to the same extent, additionally also the JD-curve shifts outward as long as $r > 0$. Hence, if $F = H$ rise simultaneously all shifts of the JC-curve cancel, while the shift in the JD-curve remains.

Proposition 2.2. *The $F = H$ scheme:* *Let $r > 0$. Then $F = H > 0$ leads to more job creation and acceptance, less job destruction and consequently reduced unemployment.*

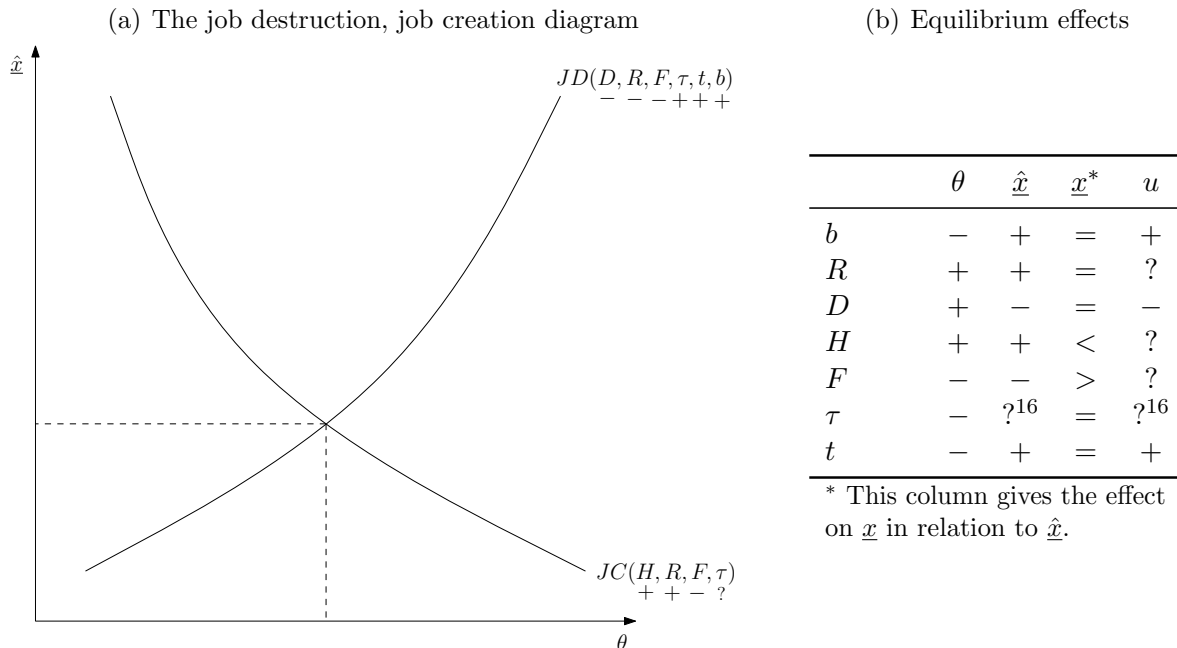
Proof. This follows directly from (2.17), (2.18), and (2.19). ■

The positive effect of this scheme, also described in Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), can be explained as follows. Looking at the life cycle of a job, a $F = H$ scheme can be compared to an interest free loan to the firm, as it gets H at the beginning of a job and eventually pays back the same amount without capital user costs. The gain is therefore reflected in the rF -term in the job destruction condition (2.18). Due to the dynamic structure of the model the alternative interpretation when considering the

¹⁴See appendix section F.2 for the analytic derivation.

¹⁵ ∇ denotes the determinant of the JD-JC system which is always positive. Ψ is the derivative of the conditional expectation with respect to the cut-off. See appendix section F.1 for details.

Figure 2.1: Uncompensated comparative statics



cross-section of firms at a specific point in time is that $F = H > 0$ implies redistribution from the firing to the hiring firms.

A recruitment subsidy affects both curves. Both shift outward leading to an increase in labor market tightness, but the JC-curve moves stronger which implies more job destruction. Compared to a hiring subsidy which is only paid if a job is created, a recruitment subsidy is received by the firm irrespective of whether a match occurs or not. The main consequence is that a hiring subsidy will partly go to the worker, while the latter subsidy is already sunk in the wage bargaining. All effects are summarized in table 2.1. These uncompensated comparative static exercises provide intuition through which channels my policy instruments work. In order to characterize optimal policy I have to develop a notion of efficiency and I have to close the government's budget constraint to restrict the analysis to policy that is implementable. This is done in the next section.

2.2 Efficiency and the optimal policy mix

Efficiency can be distorted in many ways. I will consider two possibilities: first, the typical search externality that comes from the way workers and firms are matched. Second and more at the focus my analysis, I will consider a firing externality in the spirit of BT08 stemming from the requirement to finance unemployment benefits which is not taken

¹⁶The sufficient and necessary condition for \hat{x} to increase (assuming $F = H$ and $\omega = \eta$ for simplicity) is: $[\pi^n + r + q^w G(\underline{x})] \underline{x} > [q(\theta) - \pi^n] (r + \pi^n) \Gamma$. Hence, it is also sufficient to raise u .

into account by the agents. I will concentrate on this fiscal externality and not on the problem of how unemployment compensation, which I take as given, should be optimally set. Michau (2011) explicitly models the insurance problem with risk-averse workers in a comparable setup and finds that the welfare optimum requires full insurance, i.e. $w = z$ and output maximization. As full insurance is incompatible with Nash bargained wages, I condition my efficiency analysis on the prior implementation of a partial insurance system. I assume that insurance is not perfect but that b is set such that the value of non-work z is close to the value of work¹⁷. My model with risk-neutrality can then be considered as a linear approximation of a more complex model that features concavity in the utility function (see Hagedorn and Manovskii, 2008 for a similar argumentation). The quality of the approximation naturally decreases in the difference of w and z , which as argued above, is assumed to be small. In order to analyze named inefficiencies I compute the solution to the social planner's problem of maximizing total output¹⁸ subject to the job flow constraint and the evolution of total productivity y , i.e.

$$\max_{\{x, \hat{x}, \theta\}} \Omega = \max_{\{x, \hat{x}, \theta\}} \int_0^\infty e^{-rt} (y + uh - C\theta u) dt \quad (2.21)$$

subject to

$$\dot{u} = \pi^n G(\hat{x})(L - u) - q^w u, \quad (2.22)$$

$$\dot{y} = u\theta q(\theta) \int_x^\infty \tilde{x} dG(\tilde{x}) + (L - u)\pi^n \int_{\hat{x}}^\infty \tilde{x} dG(\tilde{x}) - \pi^n y. \quad (2.23)$$

The solution to the social planner's problem is given by the following three reduced equations for socially optimal job creation, job destruction, and job acceptance:¹⁹

$$(1 - \eta) \frac{x^e - \hat{x}}{\pi^n + r} - \frac{C}{q^f} = 0, \quad (2.24)$$

$$\hat{x} + \frac{\pi^n}{\pi^n + r} \int_{\hat{x}}^\infty (\tilde{x} - \hat{x}) dG(\tilde{x}) - h - \frac{\eta}{1 - \eta} C\theta = 0, \quad (2.25)$$

$$\underline{x} = \hat{x}. \quad (2.26)$$

Comparing those relations with the decentralized equilibrium equations (2.17) to (2.19) in a policy free world, i.e. $b = F = \tau = t = D = H = R = 0$, reveals that they coincide if and only if $\omega = \eta$ (Hosios, 1990). From now on I will follow a Ramsey approach and assume that unemployment compensation $b > 0$ is exogenously given and has to be financed with

¹⁷This is also reflected in the calibration choices later on.

¹⁸In case of risk-neutral agents the solutions to the problems of maximizing output or utilitarian welfare coincide. Note that the social planner is bound to the matching technology by assumption. This is why I do not consider policy instruments that could potentially alter the matching technology itself, e.g. additional funds for employment agencies, etc.

¹⁹See appendix section D for derivation.

the least possible distortions using my instruments. Subtracting (2.24) to (2.26) from (2.17) to (2.19) gives the conditions that the policies in question have to fulfill to restore efficiency,

$$\frac{x^e - \hat{x}}{\pi^n + r} [(1 - \omega)(1 - \tau) - (1 - \eta)] + \frac{R}{q^f} = (1 - \omega)(H - F), \quad (2.27)$$

$$- \tau(\hat{x} + \pi^n \Gamma) - \frac{b + th}{1 - t} + D + rF - C\theta \left[\frac{\omega}{1 - \omega} - \frac{\eta}{1 - \eta} \right] + \frac{\omega}{1 - \omega} R\theta = 0, \quad (2.28)$$

$$F = H, \quad (2.29)$$

where $\Gamma \equiv \frac{1}{r + \pi^n} \int_{\hat{x}}^{\infty} (\tilde{x} - \hat{x}) dG(\tilde{x})$. In addition, the government's budget constraint²⁰ must be met

$$0 = (L - u)\bar{w}t + (L - u)\bar{x}\tau + q^w u(F - H) - \theta uR - (L - u)D - ub. \quad (2.30)$$

The important consequence from introducing a job acceptance margin is that $F = H$ has to hold even if the Hosios condition is not fulfilled²¹. In what follows I characterize two alternative implementations of the optimal allocation, one involving hiring and the other using wage subsidies. I depict the limitations to both schemes.

Let me first assume that the search externalities do not distort the equilibrium, i.e. $\omega = \eta$. Inserting (2.29) in (2.27) reveals that output taxation and recruiting cost subsidization are not required for efficiency, hence $\tau = R = 0$. Unemployment benefits then have to be financed using the wage tax $t = \frac{b}{\bar{w}} \frac{u}{L - u} > 0$, which is chosen to fulfill (2.30). As a compensated firing tax, $F = H$, is budget neutral, I can set F in order to fulfill (2.28), hence $F = \frac{b + th}{(1 - t)r} > 0$.

Proposition 2.3. Implementation 1a: *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is possible to implement the socially optimal allocation and balance the budget using a wage tax, $t > 0$, a firing tax and a hiring subsidy, $F = H > 0$.*

Observe the difference compared to BT08. In their framework the optimal policy consists of zero wage taxes and a firing tax to finance unemployment benefits and offset the involved distortions. Here, a firing tax will distort the acceptance margin unless a firing tax is fully compensated by a hiring subsidy. As both instruments together are budget neutral a firing tax cannot be used for financing unemployment compensation. Instead of the redistribution from the firms to the workers as in BT08, I require redistribution from

²⁰Note that in equilibrium the number of outflows $\pi^n G(\underline{x})(L - u)$ is equal to the inflows $q^w u$. Hence, $F = H$ is budget neutral in equilibrium. One should keep in mind that the introduction of a $F = H$ scheme shifts the JD-curve inward leading to more outflow out of and less inflow into unemployment. Hence, during transition the outlay on H will exceed the revenue generated by F .

²¹Because of lemma 2.1 this finding also generalizes to a framework with risk-averse workers and is independent of whether welfare or output is maximized.

employed to unemployed workers and from firing to hiring firms.

Now consider the case where $\omega \neq \eta$. Observe that at least one of the two policy instruments τ or R , is needed to satisfy equation (2.27). First I focus on output taxation, hence setting $R = 0$. The efficient output tax rate²² is then given by $\tau = 1 - \frac{1-\eta}{1-\omega}$ which is smaller than zero i.e. a subsidy if $\omega > \eta$ and positive if $\omega < \eta$. Therefore, the budget-solving wage tax rate will be higher ($\omega > \eta$) or smaller ($\omega < \eta$) compared to the benchmark tax rate where the Hosios condition holds. Again F is set to fulfill (2.28) and therefore the implementation of the optimal allocation is complete. Note that the case $F < 0$ cannot be ruled out now. Instead of τ one could alternatively use $R = \frac{x^e - \hat{x}}{\pi^n + r}(\omega - \eta)q^f$ by the same argument.

Proposition 2.4. Implementation 1b: *In case of unemployment compensation $b > 0$ and $\omega \neq \eta$ it is possible to implement the optimal allocation and balance budget using a wage tax t , a firing tax and a hiring subsidy, $F = H$, and at least one of the following two instruments: output (τ) or recruitment (R) tax/subsidy.*

MP03 do not explicitly consider the case of $\omega \neq \eta$ but it is easy to see that their job creation curve can be moved to the optimum just by adjusting $F \neq H$ accordingly. In my case this is not possible as $F = H$ is always required to offset the distortions at the job acceptance margin. Hence, the job creation curve can only be shifted by additional instruments, such as an output or a recruitment tax/subsidy.

The above implementations might require the firing tax to be of considerable magnitude. This will certainly be an issue when firms are liquidity constrained, e.g. $F \leq F_{max}$ (see BT08) which will eventually prevent the implementation of the optimal allocation. This becomes even more severe in the following extension. One can assume that F only partly improves the government's budget, say by F_{tax} as a fixed part $F_{cost} = F - F_{tax}$ reflects sunk firing costs, e.g. the administrative costs of a lay-off, etc. Obviously, $F = H$ is no longer budget neutral, implying that the wage tax t has to rise to close the budget constraint and $F = H$ have to be even higher to undo the additional distortion of the increased wage tax. Hence, it is more likely to hit F_{max} .

Note that a wage subsidy D is not required for achieving efficiency but possibly provides an alternative implementation. For simplicity assume again that $\omega = \eta$ and set $F = H = \tau = R = 0$. The lump-sum wage subsidy D , in addition to unemployment compensation b , is financed using a wage tax t , ergo $D = \bar{u}t - \frac{u}{L-u}b$. The job destruction curve will

²²Note that in a model with physical capital an output taxation would distort capital usage. This issue is ignored in this paper.

coincide with its social optimal counterpart if and only if $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L-u}b = \bar{w}t$. For $u > 0$ I can derive a necessary condition for the replacement ratio, namely $\frac{b}{\bar{w}} < \frac{1}{4}$. The contrapositive reads:

Proposition 2.5. Implementation 2: *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is **not** possible to implement the optimal allocation and balance budget using only a wage tax t and a wage subsidy D , if the replacement ratio is higher than 25%.*

Proof. $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L-u}b = \bar{w}t \xrightarrow{u>0} \frac{b}{1-t} + \frac{ht}{1-t} < \bar{w}t \Rightarrow \frac{b}{\bar{w}} < t(1-t) - t\frac{h}{\bar{w}} \Rightarrow \frac{b}{\bar{w}} < \max_{t, \frac{h}{\bar{w}} \in [0,1] \times [0,1]} [t(1-t) - t\frac{h}{\bar{w}}] = \frac{1}{4} \Leftrightarrow \frac{b}{\bar{w}} < \frac{1}{4}$ ■

This implementation is very specific to the way I introduced those instruments, i.e. D being lump-sum and t being linear. A possible implementation with $D > 0$ and $t > 0$ would mimic a progressive tax schedule. As the condition of proposition 2.5 is hardly met in any OECD economy anyways I shall focus on implementation 1 in what follows.

3 An intergroup model with economic turbulence

So far, I focused on intragroup redistribution. Allowing for intergroup redistribution enriches the model considerably because it enables me to evaluate more realistic policies. MP03 find in a numerical simulation that a wage subsidy targeted at low-skilled workers and financed by high-skilled workers works quite well in bringing down overall unemployment²³. Besides the connection via the government's budget constraint, they assume the two skill classes to operate in complete juxtaposition. The issue that "targeting is likely to damage the quality and quantity of labor supply" (Bovenberg et al., 2000) is therefore hardly addressed. The aim of this section is to show how the optimal policy mix is altered by the presence of economic turbulence and I find that a scheme as proposed by MP03 might be considerably less effective in such an environment. The idea that increased economic turbulence affects labor market outcomes is related to Ljungqvist and Sargent (1998), who assume that unemployed workers lose their skills in the course of time²⁴ as they cannot keep up to date with new production technologies. In a broader interpretation, these new production techniques and requirements emerge as a result of ongoing restructuring from manufacturing to services, spread of new information technologies, internationalization of production, etc. which all lead to expeditious changes in the economic environment, and render previous ways of production obsolete. Hence, a worker who is only familiar with

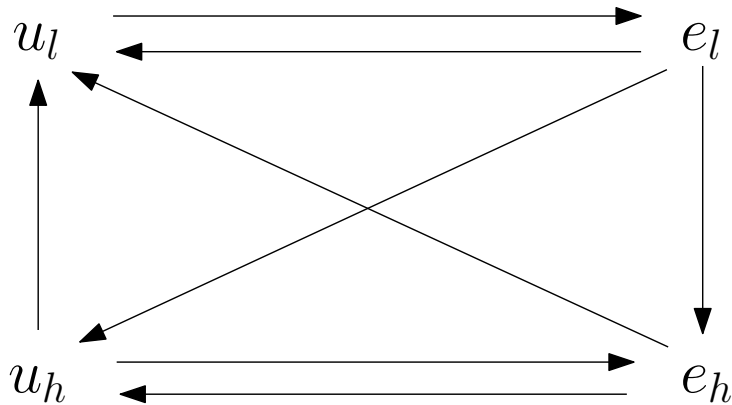
²³For the 'European calibration' they find that a 20% wage subsidy decreases low-skilled unemployment from 16.2% to 7.6% while the unemployment rate of high-skilled workers rises from 4.5% to only 4.9%.

²⁴Empirical evidence for skill loss upon separation or during unemployment, which is often approximated by the difference between the old wage and the re-employment wage, is widely documented. See for example Fallick (1996) for the U.S. and Burda and Mertens (2001) for Germany.

outdated techniques is less productive when confronted with state-of-the-art production technology.

The key differences compared to the simple intragroup model described above follow from the introduction of a second skill class with the property that the productivity distribution function of the high-skilled (h) first-order stochastically dominates the cumulative distribution function (cdf) of the low-skilled (l), i.e. $G_h(x) \leq G_l(x)$, $\forall x$. Introducing economic turbulence is modeled as follows. High-skilled workers lose their skills conditional on job loss and during unemployment with probability π^l , which means that they can only draw from $G_l(\cdot)$ when they are matched again. Low-skilled workers, on the other hand, receive a skill upgrade during employment, reflecting 'learning-on-the-job', with probability π^h , which allows them to draw a new productivity from $G_h(\cdot)$ instead of $G_l(\cdot)$. Hence, the skill composition is endogenous. For simplicity I assume that the skill of a specific worker can be observed by firms and the government at any time. Hence, a firm can direct search towards the skill class which is more profitable for the firm.²⁵ Observability of the type by the government is naturally of crucial importance to be able to target policy and a strong assumption, especially when the supports of the productivity distributions are overlapping. Limited observability would in particular affect the implementation of cross-subsidization schemes in the spirit of MP03. I will return to this later when discussing uniform policy options.

Figure 3.1: The transition flows



An individual can be in four different states, employed with high or low skills and unemployed with high or low skills, where I assume that total labor force is normalized to 1, hence: $e_l + u_l + e_h + u_h = L_l + L_h = 1$. Transitions between these states are

²⁵I abstract from undirected search of the form that firms cannot ex-ante distinguish workers by their skill and have to source from a single pool, as presented e.g. in Albrecht and Vroman (2002), as it would add an additional externality to my framework. Firms would not internalize specifically the positive effect of employment on the average quality of the pool of workers from which they source, which would lead to inefficiently low job creation.

illustrated by figure 3.1 and are formally reported in appendix section A. Note that I now additionally allow for exogenous, productivity-unrelated, separation at a rate π^x , which does not provide additional analytic insight, but is important to quantitatively match the model to the data. Beside the productivity distributions I allow high- and low-skilled workers to differ in other dimensions, like the matching technologies, as well. Differences are indicated by the subscript $j \in \{h, l\}$. All the assumptions of the intragroup model still apply unless stated otherwise. Hence, the models are nested, i.e. the intragroup model is a special case of the intergroup model with $\pi^h = \pi^l = \pi^x = 0$ and dropped skill indices. The asset value of unemployment for the low-skilled workers is the same as before, while high-skilled workers lose U_h in case they are not matched with probability π^l and only get U_l instead²⁶

$$rU_j = z_j + q_j^w (W_j^e - U_j) + \mathbb{I}_h(j)(1 - q_h^w)\pi^l (U_l - U_h). \quad (3.1)$$

The value of working²⁷ differs for both skill classes as follows. While the outside option of a low-skilled worker is only U_l , the possibility of a skill loss has to be incorporated in the outside option of a high-skilled worker, hence: $\bar{U} \equiv \pi^l U_l + (1 - \pi^l)U_h$. On the other hand only a low-skilled worker can receive a skill upgrade during work

$$\begin{aligned} rW_j(x) = & (1 - t_j)w_j(x) + \pi_j^x [\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U} - W_j(x)] \\ & + \pi_j^n \left[(1 - G_j(\hat{x}_j))\hat{W}_j^e + G_j(\hat{x}_j) (\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U}) - W_j(x) \right] \\ & + \mathbb{I}_l(j)\pi^h \left[(1 - G_h(\hat{x}_h))\hat{W}_h^e + G_h(\hat{x}_h)\bar{U} - W_l(x) \right]. \end{aligned} \quad (3.2)$$

I turn to the firms' side. As the skill of the workers can be perfectly observed, firms are able to discriminate and specifically post a vacancy for high- or low-skilled workers. A firm will enter the labor market that generates higher returns. I further assume that it can reassess this decision every period. Let me therefore define $V^m \equiv \max \{V_h, V_l\}$. The values of posting vacancies in the high- and the low-skill market, respectively, are given as

$$rV_j = -c_j + q_j^f (J_j^e + H_j - V_j) + (1 - q_j^f) (V^m - V_j), \text{ with } c_j \equiv C_j - R_j. \quad (3.3)$$

Employing a high-skilled worker yields a per-period return of rJ_h similar to before, while

²⁶ \mathbb{I} denotes the indicator function of form $\mathbb{I}_i(j) = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$.

²⁷Note that the 'inside' asset values ($\hat{W}_j(x)$ and $\hat{J}_j(x)$) are set up analogously and are not reported in the text but only in the appendix section B for the sake of completeness.

rJ_l again accounts for the possibility of a skill upgrade

$$\begin{aligned}
rJ_j(x) = & (1 - \tau_j)x - w_j(x) + D_j + \pi_j^x [(V^m - F_j) - J_j(x)] \\
& + \pi_j^n \left[(1 - G_j(\hat{x}_j)) \hat{J}_j^e + G_j(\hat{x}_j)(V^m - F_j) - J_j(x) \right] \\
& + \mathbb{I}_l(j) \pi^h \left[(1 - G_h(\hat{x}_h)) \hat{J}_h^e + G_h(\hat{x}_h)(V^m - F_h) - J_l(x) \right].
\end{aligned} \tag{3.4}$$

Wages are again determined by Nash bargaining and are related in the following way: $w_j(x) = \hat{w}_j(x) - r_j \omega (F_j - H_j)$, where $r_h = r + \pi_h^x + \pi_h^n$ and $r_l = r + \pi_l^x + \pi_l^n + \pi^h$. Wages now do not only depend on their 'own' endogenous variables and parameters but also on those of the other skill group²⁸. Importantly, this dependence is asymmetric. While wages of both skill classes increase in the own outside options, i.e. $\frac{\partial w_h}{\partial U_h} > 0$ and $\frac{\partial w_l}{\partial U_l} > 0$, as before, this is not true for the 'cross terms', i.e. $\frac{\partial w_h}{\partial U_l} > 0$ and $\frac{\partial w_l}{\partial U_h} < 0$. These derivatives capture policy spill-over of a targeting scheme. Subsidizing low-skilled workers will increase U_l and lead high-skilled workers to bargain a higher wage as their fall back option, which includes that they eventually become low-skilled, increases²⁹. By contrast, if high-skilled workers are subsidized, low-skilled workers will bargain a lower wage because working with low skills includes the increased option value of becoming high-skilled. I will now characterize the equilibrium.

3.1 Equilibrium

The equilibrium vector $\langle u_h, u_l, e_h, e_l, \theta_h, \theta_l, \underline{x}_h, \underline{x}_l, \hat{x}_h, \hat{x}_l \rangle$ is pinned down by equations (3.5) to (3.7) and the steady state flow equations (A.1), as reported in the appendix. In comparison to the intragroup model, the job creation conditions hardly change

$$\text{JC}_j : (1 - \omega) \left(\frac{(x_j^e - \hat{x}_j)(1 - \tau_j)}{r_j} - F_j + H_j \right) = \frac{c_j}{q_j^f}. \tag{3.5}$$

The job destruction conditions are now more involved. After defining $\Gamma_j \equiv \frac{1}{r_j} \int_{\hat{x}_j}^{\infty} (\tilde{x} - \hat{x}_j) dG_j(\tilde{x})$, they read

$$\begin{aligned}
\text{JD}_j : & (1 - \tau_j) \hat{x}_j - \hat{w}_j(\hat{x}_j) + D_j + rF_j - \mathbb{I}_l(j) \pi^h (F_h - F_l) \\
& + (1 - \omega) \pi_j^n (1 - \tau_j) \Gamma_j + \mathbb{I}_l(j) (1 - \omega) \pi^h (1 - \tau_h) \Gamma_h = 0.
\end{aligned} \tag{3.6}$$

²⁸See appendix section C for an explicit derivation of all four wage schedules.

²⁹Note that this effect should be even stronger if workers are risk-averse.

The relationship of the cut-off productivities, representing the job acceptance decision, is given by

$$\text{JA}_j : \underline{x}_j = \hat{x}_j + \frac{r_j}{1 - \tau_j}(F_j - H_j). \quad (3.7)$$

Analogously to before equilibrium is partly recursive. After inserting (3.7) in (3.5) in order to compute x_j^e , one can solve the remaining JD_j-JC_j system of four equations for θ_h , θ_l , \hat{x}_h , and \hat{x}_l . Knowing θ_j , \hat{x}_j , and \underline{x}_j allows to solve for e_j and u_j using (A.1).

3.2 Efficiency and the optimal policy mix

As before I start out by computing the solution to the social planner's problem, which is documented in appendix section E. Again, efficiency in a policy-free world is guaranteed if and only if $\omega = \eta$. Hence, the Hosios (1990)-condition generalizes to the complex intergroup model. I use the same Ramsey approach as before, i.e. b_h and b_l are exogenously given and have to be financed with the least possible distortion. As the implementation of the optimum should be feasible I am bound to the following budget constraint that allows for intergroup cross-subsidization

$$0 = GB_h + GB_l, \quad (3.8)$$

$$\begin{aligned} GB_j = & e_j [\bar{w}_j t_j + \bar{x}_j \tau_j - D_j] - u_j b_j - \theta_j u_j R_j - q_j^w u_j H_j \\ & + e_j \pi_j^n G_j(\hat{x}_j) F_j + e_j \pi_j^x F_j + \mathbb{I}_h(j) e_l \pi^h G_h(\hat{x}_h) F_h. \end{aligned} \quad (3.9)$$

Many insights from the intragroup model generalize to the extended model. First, $F_j = H_j$ is again a necessary condition for efficiency. Second, if $\omega = \eta$ holds I do not require output taxation τ_j or a recruiting subsidy R_j . If $\omega \neq \eta$ I need at least one of those instruments. These are important guidelines for finding an implementation of the optimal allocation for the complex intergroup model which is a non-trivial task because of several complications. First, even $b_j > 0$ and $b_{i \neq j} = 0$ requires both wage tax rates to be non-zero. Second, a $F_j = H_j$ -scheme is only budget neutral if $F_l = H_h$ as there are no taxes paid or subsidies received if a worker 'upgrades'. Consequently, whenever $F_l > H_h$ the effect on the budget is negative, if $F_l < H_h$ it is positive. If I require $F_h = F_l = H_h = H_l$ then the implementation of the efficient allocation is given by the vector $\langle t_h, t_l, F_l \rangle$ that satisfies the governments budget constraint (3.8) and pushes the JD-curves to their optimum, i.e. (C.16) minus (E.17) is equal to zero and (C.17) minus (E.16) is equal to zero. If I do not require $F_l = H_h$ then I have an additional degree of freedom and can have many optimal implementations. The central insight is that the idea of implementation 1 generalizes to the complex model. Hence, the $F = H$ -scheme is robust to the presence of economic turbulence.

3.3 Simulation

Although the theoretical treatment gives a lot of insight I will perform some numerical simulations³⁰ for two reasons. First, I provide quantitative evidence for the possible welfare gain when the optimal policy is implemented. Second, I will show that the quantitative relevance of the additional spill-over effects of targeting schemes, which arise in presence of economic turbulence, is considerable. I will do so by revisiting the effects of a cross-financed wage subsidy scheme proposed by MP03 who find that such a policy can considerably increase employment and output in their 'European calibration' case. For the sake of comparability I will also focus on Europe and target EU averages.³¹ The first task is to find a reasonable calibration for the model to fit EU labor market characteristics. I specify the functional forms of $q_j(\cdot)$ and $G_j(\cdot)$ following MP03, Den Haan et al. (2005), or Ljungqvist and Sargent (1998, 2004)

$$q_j(\theta_j) = A_j \theta_j^{-\eta}, \quad (3.10)$$

and a uniform distribution on the interval $[\underline{\kappa}_j, \bar{\kappa}_j]$

$$G_j(x) = \frac{x - \underline{\kappa}_j}{\bar{\kappa}_j - \underline{\kappa}_j}. \quad (3.11)$$

A period is chosen to be a month. Targeting an long-run interest rate³² of 3% p.a. results in $r = 0.0025$. Labor force is normalized to 1. To partition labor force into low- and high-skilled I had to use educational attainment as a proxy.³³ Being of low skill is defined as ISCED 0-2, i.e. up to lower secondary education. I target the average unemployment rate from 2004 to 2014 of 9% according to Labour Force Survey (LFS) data by Eurostat. The average share of low-skilled in total unemployment is $u_l/u = 0.38$, the share of low-skilled in total employment is $e_l/e = 0.22$. This pins down u_l , u_h , e_l and e_h . OECD Statistics report an average unemployment duration for EU-28 of 14.78 months (2004-2014). For information on skill specific duration I relied on the LFS 2009 ad-hoc module focusing on the transition from school to work as an approximation. It is reported that high-skilled search for a job only 0.52 times as long as low-skilled. This pins down the job finding rates q_h^w and q_l^w . The separation rates follow from the steady state conditions $\dot{u}_l = 0$ and $\dot{u}_h = 0$ (see appendix A). The split between endogenous ($\pi_j^n G_j(\hat{x}_j)$) and exogenous (π_j^x)

³⁰The simulations were performed using MATLAB. The code is available upon request.

³¹In general I use data for all EU-28 member states. For some data inputs I had to exclude individual countries from the averages due to data limitations.

³²I excluded the low interest years after 2007.

³³Recall that the model's distinction between low- and high-skilled is a matter of work experience and keeping up to date with new production technology, etc. rather than formal education. The later had to be chosen due to data availability.

separations is chosen to replicate an elasticity of the separation rate w.r.t. labor market tightness of -0.2 which falls into the range estimated by European Commission (2013) for 22 member states.³⁴ The probability of receiving a skill upgrade π^h was set to 0.0025, which is lower than the choice by Den Haan et al. (2005) of 0.0083 for two reasons. First, as π^h is part of the separation rate for low-skilled a too high value of π^h would imply that other contributing factors have to be calibrated to negative values. Second, this way one can demonstrate that already a small amount of turbulence is sufficient for the qualitative results described later. The probability of skill loss during unemployment π^l is pinned down by $\dot{e}_h = 0$ and close to four times larger than π^h as in Den Haan et al. (2005). European Commission (2013) estimate the elasticity of the job finding rate w.r.t. labor market tightness for 22 member states, i.e. $1 - \eta$. A weighted average results in $\eta = 0.64$. For simplicity, I abstract from inefficiencies generated by search externalities in the simulations. Hence, I set $\omega = 0.64$ in order to fulfill the Hosios (1990)-condition. As I do not interpret the average duration of a vacancy or the number of vacancies but just target the duration of unemployment I am free to choose C_h and C_l in order to normalize $\theta_h = \theta_l = 1$ ³⁵.

Without loss of generality I normalize the bounds of the productivity distribution for high-skilled to $\bar{\kappa}_h = 2$ and $\underline{\kappa}_h = 1$. I use income data from the Structure of Earnings Survey (SES) from 2010 by Eurostat to approximate differences in productivity between low- and high-skilled, the latter earning about 1.42 times as much as the former conditional on being employed. The lower bound of the low skill productivity distribution is shifted downward accordingly, i.e. $\underline{\kappa}_l = \underline{\kappa}_h/1.42$. The upper bound is set to replicate the actual income differences, i.e. $\bar{w}_h/\bar{w}_l = 1.42$. A crucial choice is the value of non-work z_j . As mentioned earlier I impose linearity in the value of non-work, hence $z_j = h + b_j$, which implies that the effects of db_j and dh are equal. I further assume that there is no skill specific difference in the value of home production. The unemployment benefits were computed based on average skill specific earnings from the SES 2010 for the individual member states using the OECD tax-benefit calculator³⁶. The weighted averages of the net replacement rates, $b_j/((1 - t_j)\bar{w}_j)$, are 0.65 for low-skilled and 0.55 for high-skilled. I set b_l and b_h accordingly. Exploiting cross-country variation, Costain and Reiter (2008) estimate the semi-elasticity of unemployment with respect to the replacement ratio $\frac{b}{w}$, i.e. $\frac{d \ln u}{d(b/w)}$ approximately in the range of [2, 3]. I set a common $h = 0.28$ to comply with the Costain and Reiter (2008)-target, namely $\frac{d \ln u}{d(b/w)} = 2.3$. In terms of average productivity the average value of

³⁴The additional degree of freedom from fitting four parameters, i.e. π_h^x , π_l^x , π_h^n and π_l^n , to three targets, i.e. the two separations rates and their average sensitivity, was used to set a comparable share of exogenous in total separation for high- and low-skilled.

³⁵This normalization is more thoroughly described in Shimer (2005).

³⁶The OECD tax-benefit calculator was applied for the year 2010 assuming a single household with no children.

non-work is $\frac{\bar{z}}{\text{av. prod}} = 0.71$ which coincides with the corresponding value derived by Hall and Milgrom (2008) for the U.S. using a completely different calibration approach relying on estimates of the Frisch elasticity. The calibration therefore addresses the argument of Hagedorn and Manovskii (2008) that the value of non-work is substantially high, but at the same time produces a realistic responsiveness of unemployment to changes in benefits. In order to finance the expenditure on b_h and b_l I set labor taxes such that government budget holds and $t_h/t_l = 1.15$ as computed using the OECD tax-benefit calculator on SES 2010 earnings again. Table G.2 in the appendix summarizes the calibration strategy. Table 3.1 reports the calibrated parameters and the results for the decentralized economy, which serves as my benchmark.

Table 3.1: Decentralized economy, benchmark

Parameters							
r	0.0025	b_h	0.7931	H_h	0.0000	$\underline{\kappa}_h$	1.0000
h	0.2800	b_l	0.6648	H_l	0.0000	$\bar{\kappa}_h$	2.0000
π_h^x	0.0041	C_h	0.5784	F_h	0.0000	$\underline{\kappa}_l$	0.7042
π_l^x	0.0038	C_l	0.1385	F_l	0.0000	$\bar{\kappa}_l$	1.1864
π_h^n	0.0150	R_h	0.0000	D_h	0.0000	A_h	0.1209
π_l^n	0.0030	R_l	0.0000	D_l	0.0000	A_l	0.1203
π^h	0.0025	τ_h	0.0000	t_h	0.0529	ω	0.6390
π^l	0.0089	τ_l	0.0000	t_l	0.0460	η	0.6390
Results							
type	θ_j	\underline{x}_j	\hat{x}_j	L_j	e_j	u_j	u_j dur.
h :	1.0000	1.2441	1.2441	0.7656	0.7098	0.0558	10.9419
l :	1.0000	0.9958	0.9958	0.2344	0.2002	0.0342	21.0421
total:	-	-	-	1.0000	0.9100	0.0900	14.7800
type	find. rate	sep. rate	rej. rate	av. prod	av. wage	tot. repl.	welfare
h :	0.0914	0.0077	0.2441	1.6221	1.5224	0.7048	1.1034
l :	0.0475	0.0062	0.6048	1.0911	1.0721	0.8813	0.2041
total:	0.0747	0.0074	-	1.5053	1.4234	-	1.3076

Note: ' u_j dur.' is unemployment duration. 'find. rate' is the job finding rate including acceptance q_j^w . 'sep. rate' is the job separation rate including exogenous and endogenous separation. 'rej. rate' is rejection rate $G_j(\underline{x}_j)$. 'av. prod' is average productivity \bar{x} . 'av. wage' is average wage \bar{w} . 'tot. repl.' is z_j/\bar{w}_j . 'welfare' is per-period in steady state. Other variables as in the paper.

In contrast, table 3.2 shows the results of the social optimum. The chosen welfare criterion increases by 5.5%. Unemployment is at 3.9% compared to 9.0%, while average duration of unemployment should optimally be close to 8 months instead of almost 15. Comparing the endogenous decision variables one can observe two things. First, reservation productivities for accepting and destructing jobs are inefficiently high, especially for the low-skilled who reject three out of five offers instead of two out of five which would be optimal. Second, job creation is inefficiently low. Again, this is more severe for low-skilled workers where market tightness is about one fourth of what it should be.

Table 3.2: Social planner's solution

Parameters							
r	0.0025	b_h	-	H_h	-	$\frac{\kappa_h}{\bar{\kappa}_h}$	1.0000
h	0.2800	b_l	-	H_l	-	$\bar{\kappa}_h$	2.0000
π_h^x	0.0041	C_h	0.5784	F_h	-	$\frac{\kappa_l}{\bar{\kappa}_l}$	0.7042
π_l^x	0.0038	C_l	0.1385	F_l	-	$\bar{\kappa}_l$	1.1864
π_h^n	0.0150	R_h	-	D_h	-	A_h	0.1209
π_l^n	0.0030	R_l	-	D_l	-	A_l	0.1203
π^h	0.0025	τ_h	-	t_h	-	ω	-
π^l	0.0089	τ_l	-	t_l	-	η	0.6390
Results							
type	θ_j	\underline{x}_j	\hat{x}_j	L_j	e_j	u_j	u_j dur.
h :	1.7916	1.0894	1.0894	0.8787	0.8470	0.0317	7.3580
l :	3.6443	0.8984	0.8984	0.1213	0.1138	0.0075	8.7280
total:	-	-	-	1.0000	0.9608	0.0392	7.6189
type	find. rate	sep. rate	rej. rate	av. prod	av. wage	tot. repl.	welfare
h :	0.1359	0.0054	0.0894	1.5447	-	-	1.2666
l :	0.1146	0.0052	0.4027	1.0424	-	-	0.1128
total:	0.1318	0.0054	-	1.4852	-	-	1.3794

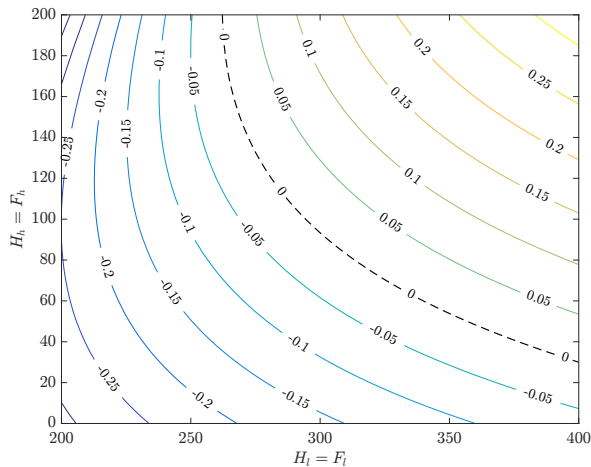
Note: See table 3.1.

Implementation of the optimal allocation

Let me now address possible implementations of the social optimum. We have learned from the previous sections that in case of $\omega = \eta$ one does not require output taxation τ_j or a recruiting subsidy R_j . Further, given proposition 2.5 for the intragroup model and the high empirical replacement ratios an implementation relying on wage subsidies does not seem to be very promising. Hence, I try to implement the corresponding intergroup variant of the policy scheme suggested in proposition 2.3. I proceed as follows. First, I set $F_j = H_j$. As $\omega = \eta$ the job creation conditions coincide with their social optimal counterparts. Given $F_j = H_j$ and b_j one can now compute the tax rates t_j that satisfy the two optimal job destruction conditions simultaneously. All the possible pairs of $F_h = H_h$ and $F_l = H_l$ that satisfy the budget constraint, i.e. set the budget surplus to 0, represent an implementation of the optimal allocation. Figure 3.2 illustrates these socially optimal combinations. Moving along the optimal isoline does not only change the combination of $F_h = H_h$ and $F_l = H_l$ but also the corresponding optimal tax rates as shown in the right table of figure 3.2. The higher $F_h = H_h$ the higher t_h has to be compared to t_l . It is also revealed that there is a first best implementation where $F_h = H_h = F_l = H_l$ are set uniformly, though this in general requires $t_h \neq t_l$. The striking result is that such schemes involve tremendously high firing taxes and hiring subsidies. To get a feeling for magnitude: the lowest possible value for $F_l = H_l$ in the table is still more than 240 times larger than the monthly wage of a low-skilled in the benchmark case. Hence, it is of interest how close one can get to the optimum if one is limited in the extend to which firing taxes and hiring

Figure 3.2: First best implementations in the intergroup model

(a) Budget surplus for efficient tax rates and all combinations of $H_j = F_j$



(b) Possible implementations of the optimal allocation

$H_h = F_h$	$H_l = F_l$	t_h	t_l
60	339.9	-0.065	0.672
80	313.3	-0.052	0.618
100	294.4	-0.041	0.562
120	281.1	-0.031	0.504
140	272.2	-0.023	0.446
160	266.5	-0.016	0.387
180	263.3	-0.010	0.326
200	262.2	-0.005	0.266
220	262.6	0.000	0.205
240	264.3	0.005	0.144
260	267.0	0.009	0.082

subsidies can be introduced as the effects presumably work in a concave way, which is discussed now.

Policy options and trade-offs

Consider the uniform policy case $F_j = H_j = 134.2$ representing a 'half-way' policy mix (*policy option 1*). Wage taxes are adjusted to satisfy the budget constraint and keep the ratio $\frac{t_h}{t_l}$ constant. Such a policy gives a welfare gain of 3.8% instead of 5.5%. Unemployment is reduced to 5.8% and unemployment duration to 9.7 months. If the scheme is set to a value of $F_j = H_j = 10$, which amounts to approximately 7 months of average wage (*policy option 2*), unemployment is still reduced by 0.4 percentage points, unemployment duration by 0.8 months, while welfare increases by 0.5%. Another finding is that other policy instrument combinations in general fail to improve welfare, unemployment and unemployment duration³⁷ simultaneously. Consider, for example, again $F_h = F_l = 10$, this time as only source of tax income and instead of using hiring subsidies, tax revenue is spent on recruitment subsidies: $R_j = 0.79 \cdot C_j$ (*policy option 3*). Such a scheme reduces overall unemployment to 6.1% and average duration of unemployment to 10.6 months, while it distorts job acceptance and creation and leaves welfare practically unchanged compared to the benchmark. In contrast if the revenue from the firing tax $F_h = F_l = 10$ is used to budget-neutrally reduce labor taxes (*policy option 4*) this implies a strong increase in welfare by 3.3%, a reduction in unemployment to 5.7% but virtually no reduction in unemployment duration. In *policy option 5* I let the hiring subsidy exceed the firing tax

³⁷The idea to focus on unemployment duration separately is that the negative effect of unemployment duration on welfare is arguably understated in the model as the per-period probability of skill loss π^l is constant while it might well be increasing in unemployment duration which might lead to severe 'lock-in' effects.

by 14% which reduces unemployment duration by almost a month while leaving welfare and unemployment unaltered. Table 3.3 summarizes the described policy options focusing on symmetric reforms, i.e. non-differentiated by skill.

Table 3.3: Various policy options in the intergroup model

policy option	F_j	H_j	R_j	Δt_h	Δt_l	Δ welf. (in %)	u (in %)	u dur.
benchmark	0.0	0.0	0	= 0	= 0	0.0	9.0	14.8
social optimum	268.4	268.4	0	< 0	> 0	5.5	3.9	7.6
policy option 1	134.2	134.2	0	< 0	< 0	3.8	5.8	9.7
policy option 2	10.0	10.0	0	< 0	< 0	0.5	8.6	14.0
policy option 3	10.0	0.0	$0.79 \cdot C_j$	= 0	= 0	0.0	6.1	10.6
policy option 4	10.0	0.0	0	< 0	< 0	3.3	5.7	14.7
policy option 5	10.0	11.4	0	> 0	> 0	0.0	9.0	13.9

Note: ΔX refers to the deviation of some variable X from the benchmark. In the five policy options labor taxes are set to balance budget while holding the ratio t_h/t_l constant in comparison to the benchmark.

Cross-financed wage subsidy schemes

I argued in the theoretical part of this section how the presence of economic turbulence can create additional spill-over effects from targeted to untargeted workers. In this section I try to quantify this for a particular targeting scheme, namely a wage subsidy for low-skilled workers financed by high-skilled workers as studied by MP03. Although a policy like that does not fulfill the criteria of being optimal, MP03 propose it as a 'better than nothing' scheme especially useful to reduce unemployment. I show that this conclusion is overthrown when economic turbulence is taken into account. To have a reference point I first replicate the MP03 result in my model when turbulence is switched off, i.e. $\pi^h = \pi^l = 0$. Hence, the skill composition of the labor force is not endogenous anymore but exogenously fixed, i.e. $L_h = 0.7656$. To replicate my targets for unemployment, its duration and composition I have to recalibrate some of the remaining transition probabilities³⁸. I then rerun the MP03 experiment by increasing D_l stepwise from 0 to 0.5. This is done in an uncompensated way and also if financed by the high-skilled workers. Table 3.4 summarizes the results.

As in MP03 a low-wage subsidy scheme seems to be very effective in reducing overall unemployment, which can be brought down to 7.38 % for $D_l = 0.5$ in the tax-compensated scenario. However, when I take economic turbulence into account, the results reverse. It is striking that even in the uncompensated case, i.e. the subsidy is given away for free, total unemployment will increase with D_l . Two effects, one boosting u_h and the other

³⁸In detail, $A_h = 0.382$, $A_l = 0.264$, $\pi_h^n = 0.0045$, $\pi_l^n = 0.0029$, $\pi_h^x = 0.0038$, and $\pi_l^x = 0.0058$. In addition, as wages slightly differ I have to set the tax rates (keeping the relative ratio constant) to $t_h = 0.045$ and $t_l = 0.039$. Again, I choose $C_h = 0.367$ and $C_l = 0.067$ in order to normalize $\theta_h = \theta_l = 1$.

Table 3.4: Effect of a low wage subsidy on unemployment rates (in %)

D_l	D_l change uncompensated						D_l change compensated by t_h					
	no turbulence			turbulence			no turbulence			turbulence		
	u_h	u_l	u	u_h	u_l	u	u_h	u_l	u	u_h	u_l	u
0.0	7.29	14.59	9.00	7.29	14.59	9.00	7.29	14.59	9.00	7.29	14.59	9.00
0.1	7.29	10.48	8.04	8.26	12.97	9.45	7.38	10.48	8.10	9.09	13.53	10.30
0.2	7.29	8.31	7.53	9.42	11.70	10.05	7.50	8.31	7.69	-	-	-
0.3	7.29	6.97	7.21	10.84	10.66	10.78	7.65	6.97	7.49	-	-	-
0.4	7.29	6.04	6.99	12.57	9.80	11.65	7.82	6.04	7.40	-	-	-
0.5	7.29	5.35	6.83	14.74	9.07	12.65	8.01	5.35	7.38	-	-	-

Note: Unemployment rates are computed in percent relative to L_j . '-' denotes break down of equilibrium.

dampening the reduction in u_l come into play. In a first direct effect a rise in D_l increases the value of working as low-skilled (W_l) and consequently the value of being unemployed (U_l) with low skills. That is where the mechanism stops in the non-turbulence framework. In my case additional indirect effects start to work. As U_l increases so does the fall back option of the high-skilled workers (\bar{U}), which will raise the reservation productivities, inflate wages and therefore reduce vacancy creation for high-skilled workers. Consequently, u_h has to rise. In a third round, as the value of being high-skilled drops in relative terms this feeds back in a negative way to the low-skilled workers as the motive of accepting a low-wage job in order to eventually become high-skilled diminishes. A direct consequence is that the skill composition in the labor force is shifted towards low-skilled workers. This is the reason why such a scheme can lead to a break down of the equilibrium even for small values of D_l if the subsidy is financed through t_h as the high-skill tax base can collapse. In conclusion, a low-wage subsidy might be useful to increase low-skill employment, but is less effective in reducing low-skill unemployment, let alone total unemployment.

4 Conclusion

A dynamic model of equilibrium unemployment and bilateral wage bargaining is used to characterize optimal labor market policy in a possibly turbulent environment. The pre-policy equilibrium is distorted by a firing externality, created by an existing potentially unemployment insurance system, along three decision margins: job creation, job acceptance, and job destruction. I apply a Ramsey approach and try to find a solution to the problem of financing exogenously fixed unemployment benefits with the least possible distortions using a rich set of policy instruments: wage, output, and firing taxes as well as wage, hiring, and recruitment subsidies. It is shown that the optimal policy mix consists of a wage tax to finance unemployment compensation and a firing tax that is offset by a hiring subsidy. The latter part can be interpreted as redistribution from firing to hiring firms and helps to undo the distortions created by the wage tax system. The reason is that a 'firing tax equal hiring subsidy'-scheme, while not distorting job acceptance and job creation,

leads to less job destruction as such a policy represents an interest free loan to the firm. The derived optimal policy mix deviates from the static framework results of Blanchard and Tirole (2008) who argue that benefits should be completely financed through firing taxes. This idea does not completely transfer to my dynamic set-up. In any case a firing tax has to be compensated one-for-one by a hiring subsidy to prevent distortions along the job acceptance margin. Hence, in the case of 'unbalanced' search externalities that distort job creation, the failure of the Hosios condition cannot be corrected by a spread between the firing tax and the hiring subsidy. Instead either an output or a recruitment tax/subsidy have to be used in addition.

The important feature of the derived policy mix is that it is robust to the introduction of economic turbulence in the interpretation of Ljungqvist and Sargent (1998), i.e. skill loss during unemployment. This is crucial as a lot of existing policy advice is rendered considerably less effective in that case. I demonstrate this by reassessing a cross-financed wage subsidy scheme for low-skilled workers as, for example, suggested by Mortensen and Pissarides (2003). While they assume skill classes to operate in complete juxtaposition, except for the connection via the government's budget constraint, possible skill loss during unemployment implies that high-skilled workers become pickier concerning their acceptance and continuation decision as their fall back option, including subsidized low-skill employment, increases. The skill composition deteriorates as a result of such a targeting scheme and the finding of Mortensen and Pissarides (2003) that unemployment can be considerably reduced is overthrown. In conclusion, the paper argues that instead of redistribution from firing firms to unemployed workers (Blanchard and Tirole, 2008) or from high- to low-skilled workers (Mortensen and Pissarides, 2003), a scheme involving redistribution from firing to hiring firms should be preferred.

References

- ALBRECHT, J. AND S. VROMAN (2002): "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283–305.
- ANGRIST, J. D., G. W. IMBENS, AND D. B. RUBIN (1996): "Identification of Causal Effects Using Instrumental Variables," *Journal of the American Statistical Association*, 91, 444–455.
- BELL, B., R. BLUNDELL, AND J. REENEN (1999): "Getting the Unemployed Back to Work: The Role of Targeted Wage Subsidies," *International Tax and Public Finance*, 6, 339–360.

- BLANCHARD, O. J. AND J. TIROLE (2008): “The Joint Design of Unemployment Insurance and Employment Protection: A first pass,” *Journal of the European Economic Association*, 6(1), 45–77.
- BONIN, H., W. KEMPE, AND H. SCHNEIDER (2002): “Household Labor Supply Effects of Low-Wage Subsidies in Germany,” IZA Discussion Papers 637, Institute for the Study of Labor (IZA).
- BOOCKMANN, B., T. ZWICK, A. AMMERMÜLLER, AND M. MAIER (2007): “Do hiring subsidies reduce unemployment among the elderly? Evidence from two natural experiments,” ZEW Discussion Papers 07-001, ZEW - Zentrum für Europäische Wirtschaftsforschung / Center for European Economic Research.
- BOONE, J. AND J. C. VAN OURS (2004): “Effective Active Labor Market Policies,” IZA Discussion Papers 1335, Institute for the Study of Labor (IZA).
- BOVENBERG, L. A., J. J. GRAAFLAND, AND R. A. DE MOOIJ (2000): “Tax Reform and the Dutch Labor Market An Applied General Equilibrium Approach,” *Journal of Public Economics*, 78, 194–214.
- BROWN, A. J. G., C. MERKL, AND D. J. SNOWER (2007): “Comparing the Effectiveness of Employment Subsidies,” CEPR Discussion Papers 6334, Centre for Economic Policy Research.
- BUNDESAGENTUR FÜR ARBEIT (2008): “Arbeitsmarkt in Deutschland Zeitreihen bis 2007,” Analytikreport der Statistik.
- BURDA, M. C. AND A. MERTENS (2001): “Estimating wage losses of displaced workers in Germany,” *Labour Economics*, 8, 15–41.
- BURDA, M. C. AND C. WYPLOSZ (1994): “Gross worker and job flows in Europe,” *European Economic Review*, 38, 1287–1315.
- CAHUC, P. AND T. L. BARBANCHON (2010): “Labor market policy evaluation in equilibrium: Some lessons of the job search and matching model,” *Labour Economics*, 17, 196–205.
- CARDULLO, G. AND B. VAN DER LINDEN (2006): “Employment Subsidies and Substitutable Skills: An Equilibrium Matching Approach,” IZA Discussion Papers 2073, Institute for the Study of Labor (IZA).
- COSTAIN, J. S. AND M. REITER (2008): “Business Cycle, Unemployment Insurance, and the Calibration of Matching Models,” *Journal of Economic Dynamics and Control*, 32, 1120–1155.

- DEN HAAN, W. J., C. HAEFKE, AND G. RAMEY (2005): “Turbulence and Unemployment in a Job Matching Model,” *Journal of European Economic Association*, 3, 1360–1385.
- EUROPEAN COMMISSION (2013): “Labour Market Developments in Europe 2013,” *European Economy* 6/2013.
- FAHR, R. AND U. SUNDE (2001): “Disaggregate Matching Functions,” IZA Discussion Papers 335, Institute for the Study of Labor (IZA).
- FALLICK, B. C. (1996): “A review of the recent empirical literature on displaced workers,” *Industrial and Labor Relations Review*, 50, 5–16.
- FELDSTEIN, M. S. (1976): “Temporary Layoffs in the Theory of Unemployment,” *Journal of Political Economy*, 84, 937–957.
- HAGEDORN, M. AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706.
- HALL, R. E. (2005): “The Amplification of Unemployment Fluctuations through Self-Selection,” NBER Working Paper 11186, National Bureau of Economic Research.
- HALL, R. E. AND P. R. MILGROM (2008): “The Limited Influence of Unemployment on the Wage Bargain,” *American Economic Review*, 98, 1653–1674.
- HEIJDR, B. J. AND J. E. LIGTHART (2002): “The hiring subsidy cum firing tax in a search model of unemployment,” *Economics Letters*, 75, 97–108.
- HOSIOS, A. J. (1990): “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57(2), 279–298.
- KATZ, L. F. (1996): “Wage Subsidies for the Disadvantaged,” NBER Working Papers 5679, National Bureau of Economic Research, Inc.
- LAZEAR, E. P. (1990): “Job Security Provisions and Employment,” *Quarterly Journal of Economics*, 105, 699–726.
- LIGTHART, J. E. AND B. J. HEIJDR (2000): “Deposit-Refund on Labor - A Solution to Equilibrium Unemployment?” IMF Working Papers 00/9, International Monetary Fund.
- LJUNGQVIST, L. AND T. J. SARGENT (1998): “The European Unemployment Dilemma,” *Journal of Political Economy*, 106, 514–550.
- (2004): “European Unemployment and Turbulence Revisited in a Matching Model,” *Journal of the European Economic Association*, 2, 456–467.

- MICHAU, J.-B. (2011): “Optimal Labor Market Policy with Search Frictions and Risk-Averse Workers,” Mimeo.
- MORTENSEN, D. T. AND C. A. PISSARIDES (1994): “Job Creation and Job Destruction in a Theory of Unemployment,” *Review of Economic Studies*, 61, 397–415.
- (2003): “Taxes, Subsidies and Equilibrium Market Outcomes, in: Phelps, E. Designing Inclusion: Tools to raise low-end pay and employment in private enterprise,” Cambridge University Press.
- OSKAMP, F. AND D. J. SNOWER (2008): “The Effect of Low-Wage Subsidies on Skills and Employment,” Kiel Working Papers 1292, Kiel Institute for the World Economy.
- PISSARIDES, C. A. (1992): “Loss of Skill During Unemployment and the Persistence of Employment Shocks,” *Quarterly Journal of Economics*, 107, 1371–1391.
- (2000): *Equilibrium Unemployment Theory*, MIT Press.
- RICCI, A. AND R. WALDMANN (2011): “Job security and training: the case of Pareto improving firing taxes,” Research Paper Series 2011, Centre for Economic and International Studies.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- SINN, H.-W., C. HOLZNER, W. MEISTER, W. OCHEL, AND M. WERDING (2006): “Aktivierende Sozialhilfe 2006: Das Kombilohn-Modell des ifo Instituts,” Sonderdruck aus ifo Schnelldienst, No. 2.
- SKEDINGER, P. (2011): “Employment Consequences of Employment Protection Legislation,” Working Paper Series 865, Research Institute of Industrial Economics.
- STATISTISCHES BUNDESAMT (2007): “Statistisches Jahrbuch 2007: Für die Bundesrepublik Deutschland,” <http://www.destatis.de/>.
- TOPEL, R. H. (1983): “On Layoffs and Unemployment Insurance,” *American Economic Review*, 73, 541–559.

Appendix

A Laws of motion

$$\begin{aligned}
\dot{u}_h &= e_h [\pi_h^x + \pi_h^n G_h(\hat{x}_h)] (1 - \pi^l) + e_l \pi^h G_h(\hat{x}_h) (1 - \pi^l) \\
&\quad - u_h [(1 - q_h^w) \pi^l + q_h^w], \\
\dot{e}_l &= (1 - e_h - e_l - u_h) q_l^w - e_l [\pi_l^x + \pi_l^n G_l(\hat{x}_l) + \pi^h], \\
\dot{e}_h &= e_l \pi^h (1 - G_h(\hat{x}_h)) + u_h q_h^w - e_h [\pi_h^x + \pi_h^n G_h(\hat{x}_h)], \\
\dot{y}_h &= -y_h (\pi_h^x + \pi_h^n) + e_h \pi_h^n \tilde{G}_h(\hat{x}_h) + e_l \pi^h \tilde{G}_h(\hat{x}_h) \\
&\quad + u_h \theta_h q_h(\theta_h) \tilde{G}_h(\underline{x}_h), \\
\dot{y}_l &= -y_l (\pi_l^x + \pi_l^n + \pi^h) + e_l \pi_l^n \tilde{G}_l(\hat{x}_l) \\
&\quad + (1 - u_h - e_l - e_h) \theta_l q_l(\theta_l) \tilde{G}_l(\underline{x}_l),
\end{aligned} \tag{A.1}$$

where the partial expectation is defined as $\tilde{G}_j(x) = \int_x^\infty \tilde{x} dG_j(\tilde{x})$. Equilibrium states are derived by setting the left hand sides to zero.

B Unreported value functions and Nash bargaining

Unreported value functions:

$$\begin{aligned}
r\hat{W}_j(x) &= (1 - t_j) \hat{w}_j(x) + \pi_j^x [\mathbb{I}_l(j) U_l + \mathbb{I}_h(j) \bar{U} - \hat{W}_j(x)] \\
&\quad + \pi_j^n [(1 - G_j(\hat{x}_j)) \hat{W}_j^{\hat{e}} + G_j(\hat{x}_j) (\mathbb{I}_l(j) U_l + \mathbb{I}_h(j) \bar{U}) - \hat{W}_j(x)] \\
&\quad + \mathbb{I}_l(j) \pi^h [(1 - G_h(\hat{x}_h)) \hat{W}_h^{\hat{e}} + G_h(\hat{x}_h) \bar{U} - \hat{W}_l(x)],
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
r\hat{J}_j(x) &= (1 - \tau_j) x - \hat{w}_j(x) + D_j + \pi_j^x [(V^m - F_j) - \hat{J}_j(x)] \\
&\quad + \pi_j^n [(1 - G_j(\hat{x}_j)) \hat{J}_j^{\hat{e}} + G_j(\hat{x}_j) (V^m - F_j) - \hat{J}_j(x)] \\
&\quad + \mathbb{I}_l(j) \pi^h [(1 - G_h(\hat{x}_h)) \hat{J}_h^{\hat{e}} + G_h(\hat{x}_h) (V^m - F_h) - \hat{J}_l(x)].
\end{aligned} \tag{B.2}$$

Nash bargaining implies:

$$W_h - \bar{U} = \frac{\omega(1 - t_h)}{1 - \omega} (J_h + H_h) \quad \text{and} \quad \hat{W}_h - \bar{U} = \frac{\omega(1 - t_h)}{1 - \omega} (\hat{J}_h + F_h), \tag{B.3}$$

$$W_l - U_l = \frac{\omega(1 - t_l)}{1 - \omega} (J_l + H_l) \quad \text{and} \quad \hat{W}_l - U_l = \frac{\omega(1 - t_l)}{1 - \omega} (\hat{J}_l + F_l), \tag{B.4}$$

or

$$W_h - \bar{U} = \tilde{\omega}_h s_h \quad \text{and} \quad \hat{W}_h - \bar{U} = \tilde{\omega}_h \hat{s}_h, \tag{B.5}$$

$$W_l - U_l = \tilde{\omega}_l s_l \quad \text{and} \quad \hat{W}_l - U_l = \tilde{\omega}_l \hat{s}_l, \quad (\text{B.6})$$

$$J_j + H_j = \tilde{\omega}_j s_j \quad \text{and} \quad \hat{J}_j + F_j = \tilde{\omega}_j \hat{s}_j, \quad (\text{B.7})$$

where

$$\tilde{\omega}_j \equiv \frac{\omega(1-t_j)}{1-\omega t_j} \quad \text{and} \quad \tilde{\omega}_j \equiv \frac{1-\omega}{1-\omega t_j} \quad \text{and} \quad (1-\omega)\tilde{\omega}_j = \omega(1-t_j)\tilde{\omega}_j.$$

C Derivation of the equilibrium conditions

This section formally derives the equilibrium conditions for the intergroup model. As the model is nested, the conditions for the simple intragroup model can be found by dropping the skill index and setting $\pi^h = \pi^l = \pi^x = 0$. Let me first define $\tilde{r} = r + \pi^l$, $r_h = r + \pi_h^x + \pi_h^n$, and $r_l = r + \pi_l^x + \pi_l^n + \pi^h$. Equilibrium is determined by the free entry conditions (C.1) and the cut-off conditions (C.2), i.e.

$$V_l = V_h = 0 \quad \Rightarrow \quad V^m = 0 \quad \Rightarrow \quad J_h^e = \frac{c_h}{q_h} - H_h \quad \text{and} \quad J_l^e = \frac{c_l}{q_l} - H_l, \quad (\text{C.1})$$

$$\hat{J}_j(\hat{x}_j) + F_j = 0 \Rightarrow \hat{x}_j \quad \text{and} \quad J_j(\underline{x}_j) + H_j = 0 \Rightarrow \underline{x}_j. \quad (\text{C.2})$$

Take conditional expectation of (B.4), insert in (3.1) and eliminate J_l^e using the free entry condition (C.1) to get

$$rU_l = z_l + \frac{\omega(1-t_l)}{1-\omega} c_l \theta_l. \quad (\text{C.3})$$

Proceeding analogously for U_h results in

$$rU_h = z_h + \frac{\omega(1-t_h)}{1-\omega} c_h \theta_h - \pi^l (U_h - U_l). \quad (\text{C.4})$$

I use C.4 and C.3 to solve for the difference in the values of unemployment

$$U_h - U_l = \frac{z_h - z_l}{\tilde{r}} + \frac{\omega}{1-\omega} \frac{(1-t_h)c_h \theta_h - (1-t_l)c_l \theta_l}{\tilde{r}}. \quad (\text{C.5})$$

Wages

To get the wage equations proceed as follows. Multiplying (B.3) by r and rearranging gives $\omega(1-t_h)r\hat{J}_h(x) - (1-\omega)r\hat{W}_h(x) = -(1-\omega)r\bar{U} - \omega(1-t_h)rF_h$. Replace $r\hat{W}_h(x)$ and $r\hat{J}_h(x)$ by (B.1) and (B.2). Most of the remaining values cancel out after eliminating them using the first-order conditions from the Nash bargaining (B.5) to (B.7), and their conditional expectations. Solving for $\hat{w}_h(x)$ gives

$$\hat{w}_h(x) = \omega [(1-\tau_h)x + D_h + rF_h] + \frac{1-\omega}{1-t_h} r\bar{U}. \quad (\text{C.6})$$

Eliminating the remaining values of being unemployed, realizing that $r\bar{U} = r(1 - \pi^l)[U_h - U_l] + rU_l$, results in

$$\begin{aligned}\hat{w}_h(x) = & \frac{1 - \omega}{1 - t_h} \left(\frac{(1 - \pi^l)r}{\tilde{r}} z_h + \frac{\pi^l(1 + r)}{\tilde{r}} z_l \right) + \omega [(1 - \tau_h)x + D_h + rF_h] \\ & + \omega \left[\frac{(1 - \pi^l)r}{\tilde{r}} c_h \theta_h + \frac{1 - t_l}{1 - t_h} \frac{\pi^l(1 + r)}{\tilde{r}} c_l \theta_l \right].\end{aligned}\quad (\text{C.7})$$

The derivation of the 'outside' wage works analogously and results in $w_h(x) = \hat{w}_h(x) - r_h \omega (F_h - H_h)$ given

$$w_h(x) = \omega [(1 - \tau_h)x + D_h - (\pi_h^x + \pi_h^n)F_h + r_h H_h] + \frac{1 - \omega}{1 - t_h} r\bar{U}.\quad (\text{C.8})$$

I proceed the same way to get $\hat{w}_l(x)$ and $w_l(x)$.

$$\begin{aligned}\hat{w}_l(x) = & \omega [(1 - \tau_l)x + D_l - \pi^h(F_h - F_l) + rF_l] \\ & + \frac{1 - \omega}{1 - t_l} [rU_l - \pi^h(1 - \pi^l)(U_h - U_l)] \\ & + \omega \pi^h(1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \tilde{\omega}_h \hat{S}_h^{\hat{e}}.\end{aligned}\quad (\text{C.9})$$

Note that in case of $t_l \neq t_h$, $\tilde{\omega}_h \hat{S}_h^{\hat{e}}$ does not drop out and is replaced by $\frac{c_h}{q_h^f} + \tilde{\omega}_h \Sigma$. See below for the derivation. Eliminating the values of being unemployed gives

$$\begin{aligned}\hat{w}_l(x) = & - \frac{1 - \omega}{1 - t_l} \pi^h(1 - \pi^l) \left[\frac{z_h - z_l}{\tilde{r}} + \frac{\omega}{1 - \omega} \frac{(1 - t_h)c_h \theta_h - (1 - t_l)c_l \theta_l}{\tilde{r}} \right] \\ & + \omega [(1 - \tau_l)x + D_l - \pi^h(F_h - F_l) + rF_l + c_l \theta_l] + \frac{1 - \omega}{1 - t_l} z_l \\ & + \omega \pi^h(1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \left[\frac{c_h}{q_h^f} + \tilde{\omega}_h \Sigma \right].\end{aligned}\quad (\text{C.10})$$

Similar to before the 'outside' wage is given by $w_l(x) = \hat{w}_l(x) - r_l \omega (F_l - H_l)$ knowing that

$$\begin{aligned}w_l(x) = & \omega [(1 - \tau_l)x + D_l - \pi^h F_h - (\pi_l^x + \pi_l^n)F_l + r_l H_l] \\ & + \frac{1 - \omega}{1 - t_l} [rU_l - \pi^h(1 - \pi^l)(U_h - U_l)] \\ & + \omega \pi^h(1 - G_h(\hat{x}_h)) \frac{t_h - t_l}{1 - t_l} \tilde{\omega}_h \hat{S}_h^{\hat{e}}.\end{aligned}\quad (\text{C.11})$$

For the derivation of Σ I start out by noting that the surplus functions are linear of form $S_h(x) = S_h^0 + S_h^1 x$, and further

$$\hat{S}_h(x) = S_h(x) + (1 - \omega t_h)(F_h - H_h) = S_h^0 + S_h^1 x + (1 - \omega t_h)(F_h - H_h),\quad (\text{C.12})$$

as will be established below. Taking conditional expectation gives

$$\begin{aligned}\hat{S}_h^e &= \int_{\hat{x}_h}^{\infty} \left[\frac{S_h^0 + S_h^1 \tilde{x} + (1 - \omega t_h)(F_h - H_h)}{1 - G_h(\hat{x}_h)} \right] dG_h(\tilde{x}) \\ &= S_h^0 + (1 - \omega t_h)(F_h - H_h) + S_h^1 \frac{\tilde{G}(\hat{x}_h)}{1 - G_h(\hat{x}_h)}.\end{aligned}\tag{C.13}$$

Taking conditional expectation of $S_h(x) = S_h^0 + S_h^1 x$, eliminating S_h^0 by using (C.13) and inserting for S_h^1 establishes $\hat{S}_h^e = S_h^e + \Sigma$. Combine (C.1) and (B.7) to get $S_h^e = \frac{c_h}{q_h^f} \frac{1}{\tilde{\omega}_h}$ which gives $\tilde{\omega}_h \hat{S}_h^e = \frac{c_h}{q_h^f} + \tilde{\omega}_h \Sigma$, with

$$\Sigma = \underbrace{\frac{(1 - \tau_h)(1 - \omega t_h)}{r_h}}_{S_h^1} \left[\frac{\tilde{G}_h(\hat{x}_h)}{1 - G_h(\hat{x}_h)} - \frac{\tilde{G}_h(\underline{x}_h)}{1 - G_h(\underline{x}_h)} \right] + \underbrace{(1 - \omega t_h)(F_h - H_h)}_{\hat{S}_h(x) - S_h(x)}.$$

Note that $\Sigma = 0$ if $F_h = H_h$ because it also implies $\underline{x}_h = \hat{x}_h$ as I will prove below.

Job creation conditions

The job creation conditions are derived as follows. Subtract (B.2) evaluated at \hat{x}_j from (3.4) and replace $\hat{J}_j(\hat{x})$ by $-F_j$ using (C.2). Taking conditional expectation w.r.t. \underline{x}_j and replacing J_j^e using (C.1) gives the *job creation curves*

$$\text{JC}_j : (1 - \omega) \left(\frac{(x_j^e - \hat{x}_j)(1 - \tau_j)}{r_j} - F_j + H_j \right) = \frac{c_j}{q_j^f}.\tag{C.14}$$

Job destruction conditions

First define: $\Gamma_j \equiv \frac{1}{r_j} \int_{\hat{x}_j}^{\infty} (\tilde{x} - \hat{x}_j) dG_j(\tilde{x})$. Subtract (B.2) evaluated at \hat{x}_j from themselves and eliminate $\hat{J}_j(\hat{x})$ by $-F_j$ again using (C.2). Use the conditional expectation w.r.t. \hat{x}_j of the resulting expressions $\hat{J}_h(x)$ and $\hat{J}_l(x)$ to eliminate \hat{J}_j^e in (B.2). Evaluate again at \hat{x}_j and make use of (C.2) to arrive at

$$\begin{aligned}\text{JD}_j : (1 - \tau_j)\hat{x}_j - \hat{w}_j(\hat{x}_j) + D_j + rF_j - \mathbb{I}_l(j)\pi^h(F_h - F_l) \\ + (1 - \omega)\pi_j^n(1 - \tau_j)\Gamma_j + \mathbb{I}_l(j)(1 - \omega)\pi^h(1 - \tau_h)\Gamma_h = 0.\end{aligned}\tag{C.15}$$

Eliminating the wages and diving by $(1 - \omega)$ then gives the final *job destruction curves*

$$\begin{aligned}(1 - \tau_h)\hat{x}_h + D_h + rF_h - \frac{1}{1 - t_h} \left[\frac{(1 - \pi^l)r}{\tilde{r}} z_h + \frac{\pi^l(1 + r)}{\tilde{r}} z_l \right] \\ - \frac{\omega}{1 - \omega} \left[\frac{(1 - \pi^l)r}{\tilde{r}} c_h \theta_h + \frac{1 - t_l}{1 - t_h} \frac{\pi^l(1 + r)}{\tilde{r}} c_l \theta_l \right] + \pi_h^n(1 - \tau_h)\Gamma_h = 0,\end{aligned}\tag{C.16}$$

$$\begin{aligned}
& (1 - \tau_l)\hat{x}_l + D_l + rF_l - \pi^h(F_h - F_l) - \frac{z_l}{1 - t_l} - \frac{\omega}{1 - \omega}c_l\theta_l \\
& + \frac{\pi^h(1 - \pi^l)}{1 - t_l} \frac{z_h - z_l}{\tilde{r}} + \pi^h(1 - \pi^l) \frac{\omega}{1 - \omega} \left[\frac{1 - t_h}{1 - t_l} \frac{c_h\theta_h}{\tilde{r}} - \frac{c_l\theta_l}{\tilde{r}} \right] \\
& - \pi^h(1 - G_h(\hat{x}_h)) \frac{\omega}{1 - \omega} \frac{t_h - t_l}{1 - t_l} \left[\frac{c_h}{q_h^f} + \tilde{\omega}_h \Sigma \right] \\
& + \pi_l^n(1 - \tau_l)\Gamma_l + \pi^h(1 - \tau_h)\Gamma_h = 0.
\end{aligned} \tag{C.17}$$

Cut-off relationships

The relation between the reservation productivities \underline{x}_j and \hat{x}_j stems from a simple observation. The cut-off conditions in (C.2) in combination with (B.3) and (B.4) imply that firms and workers will always mutually agree on creating and destroying jobs. Hence, \underline{x} and \hat{x} set the joint surpluses to 0. The surpluses in equilibrium are given by

$$S_h(x) = W_h(x) + J_h(x) - \bar{U} + H_h \quad \text{and} \quad S_l(x) = W_l(x) + J_l(x) - U_l + H_l, \tag{C.18}$$

$$\hat{S}_h(x) = \hat{W}_h(x) + \hat{J}_h(x) - \bar{U} + F_h \quad \text{and} \quad \hat{S}_l(x) = \hat{W}_l(x) + \hat{J}_l(x) - U_l + F_l. \tag{C.19}$$

Note that by lemma 2.1 both surplus functions and hence cut-offs coincide if $F_j = H_j$, a result which even holds in a more general framework with non-linear utility and non-linear wage tax. Given my assumptions, observe that for the same x the difference between the surplus functions is given by $S_j(x) - \hat{S}_j(x) = \frac{-t_j}{r_j}(w_j(x) - \hat{w}_j(x)) + H_j - F_j = -(1 - \omega t_j)(F_j - H_j)$, which is independent of x . Hence, the surplus functions have the following linear structure

$$S_j(x) = S_j^0 + S_j^1 x - (1 - \omega t_j)(F_j - H_j), \tag{C.20}$$

$$\hat{S}_j(x) = S_j^0 + S_j^1 x. \tag{C.21}$$

From (3.2) and (3.4) I infer that $S_j^1 = \frac{(1 - \tau_j)(1 - \omega t_j)}{r_j}$. The cut-offs solving $S_j(\underline{x}) = 0$ and $S_j(\hat{x}) = 0$ are therefore given by

$$\underline{x}_j = -\frac{S_j^0 - (1 - \omega t_j)(F_j - H_j)}{S_j^1}, \tag{C.22}$$

$$\hat{x}_j = -\frac{S_j^0}{S_j^1}. \tag{C.23}$$

Hence, the *relationship of the cut-offs* can be written as

$$\text{JA}_j \quad : \quad \underline{x}_j = \hat{x}_j + \frac{r_j}{1 - \tau_j}(F_j - H_j). \tag{C.24}$$

D Social planner's optimum in the simple intragroup model

The constrained social optimum is derived by maximizing the social welfare function $\Omega(\cdot)$ subject to the matching constraints and the evolution of total production y , hence

$$\max_{\{\underline{x}, \hat{x}, \theta\}} \Omega = \max_{\{\underline{x}, \hat{x}, \theta\}} \int_0^\infty e^{-rt} (y + uh - C\theta u) dt \quad (\text{D.1})$$

subject to

$$\dot{u} = \pi^n G(\hat{x})(L - u) - q^w u, \quad (\text{D.2})$$

$$\dot{y} = u\theta q(\theta) \int_{\underline{x}}^\infty \tilde{x} dG(\tilde{x}) + (L - u)\pi^n \int_{\hat{x}}^\infty \tilde{x} dG(\tilde{x}) - \pi^n y. \quad (\text{D.3})$$

I set up the present-value Hamiltonian

$$\begin{aligned} \mathcal{H} = & e^{-rt} (y + uh - C\theta u) + \lambda_1 [\pi^n G(\hat{x})(L - u) - q^w u] \\ & + \lambda_2 \left[u\theta q(\theta) \int_{\underline{x}}^\infty \tilde{x} dG(\tilde{x}) + (L - u)\pi^n \int_{\hat{x}}^\infty \tilde{x} dG(\tilde{x}) - \pi^n y \right]. \end{aligned} \quad (\text{D.4})$$

The optimality conditions, i.e. $\frac{\partial \mathcal{H}}{\partial \underline{x}} = 0$, $\frac{\partial \mathcal{H}}{\partial \hat{x}} = 0$, $\frac{\partial \mathcal{H}}{\partial \theta} = 0$, $\frac{\partial \mathcal{H}}{\partial u} = -\dot{\lambda}_1$, $\frac{\partial \mathcal{H}}{\partial y} = -\dot{\lambda}_2$, imply (D.5) to (D.9), i.e.

$$\lambda_1 - \underline{x}\lambda_2 = 0, \quad (\text{D.5})$$

$$\lambda_1 - \hat{x}\lambda_2 = 0. \quad (\text{D.6})$$

From (D.5) and (D.6) one can infer that the cut-off productivities irrespective of whether one arrives at or has already been in a job coincide, i.e. $\underline{x} = \hat{x}$. From now on I will just use \underline{x} . Before stating the remaining first-order conditions define $\tilde{G}(\underline{x}) \equiv \int_{\underline{x}}^\infty \tilde{x} dG(\tilde{x})$ and $\Gamma \equiv \frac{1}{r + \pi^n} \int_{\underline{x}}^\infty (\tilde{x} - \underline{x}) dG(\tilde{x})$,

$$-e^{-rt}C - \lambda_1(1 - \eta)q(\theta)(1 - G(\underline{x})) + \lambda_2(1 - \eta)q(\theta)\tilde{G}(\underline{x}) = 0, \quad (\text{D.7})$$

$$-e^{-rt}(h - C\theta) - \lambda_1[\pi^n G(\underline{x}) + q^w] + \lambda_2[\theta q(\theta) - \pi^n]\tilde{G}(\underline{x}) = -\dot{\lambda}_1, \quad (\text{D.8})$$

$$e^{-rt} - \pi^n \lambda_2 = -\dot{\lambda}_2. \quad (\text{D.9})$$

Eliminating λ_1 in (D.7) using (D.5) gives

$$-e^{-rt}C + \lambda_2(1 - \eta)q(\theta)(r + \pi^n)\Gamma = 0, \quad (\text{D.10})$$

which implies the following relationships for λ_1 and λ_2

$$\lambda_1 = \frac{e^{-rt}C\underline{x}}{(1-\eta)q(\theta)(r+\pi^n)\Gamma} \quad \text{and} \quad \lambda_2 = \frac{e^{-rt}C}{(1-\eta)q(\theta)(r+\pi^n)\Gamma}. \quad (\text{D.11})$$

Differentiating (D.10) w.r.t. t and subtracting (D.10) again results in the following relations

$$\dot{\lambda}_2 = -\lambda_2 r \quad \text{and consequently} \quad \dot{\lambda}_1 = -\lambda_1 r. \quad (\text{D.12})$$

Inserting for λ_2 and $\dot{\lambda}_2$ in (D.9) and rearranging gives the reduced optimality condition that has a similar structure compared the job creation condition

$$(1-\eta)\frac{x^e - \underline{x}}{\pi^n + r} - \frac{C}{q^f} = 0. \quad (\text{D.13})$$

To derive the last reduced optimality condition, i.e. the job destruction condition counterpart, I eliminate λ_1 , λ_2 and $\dot{\lambda}_1$ in (D.8) and rearrange

$$\underline{x} - h + \pi^n \Gamma - \frac{\eta}{1-\eta} C \theta = 0 \quad (\text{D.14})$$

E Social planner's optimum in the intergroup model

Again I maximize discounted social welfare

$$\int_0^\infty e^{-rt} [y_h + y_l + (u_h + u_l)h - u_h C_h \theta_h - u_l C \theta_l] dt \quad (\text{E.1})$$

where $u_l = (1 - u_h - e_h - e_l)$, subject to the evolution of the employment states \dot{u}_h , \dot{e}_l , \dot{e}_h and of total production \dot{y}_h and \dot{y}_l as given by (A.1), over the choice variables \underline{x}_j , \hat{x}_j and θ_j . I set up the present-value Hamiltonian

$$\begin{aligned} \mathcal{H} = & e^{-rt} [y_h + y_l + (1 - e_h - e_l)h - u_h C_h \theta_h - (1 - u_h - e_h - e_l)C \theta_l] \\ & + \lambda_1 \dot{u}_h + \lambda_2 \dot{e}_l + \lambda_3 \dot{e}_h + \lambda_4 \dot{y}_h + \lambda_5 \dot{y}_l. \end{aligned} \quad (\text{E.2})$$

The optimality conditions $\frac{\partial \mathcal{H}}{\partial \underline{x}_j} = 0$, $\frac{\partial \mathcal{H}}{\partial \hat{x}_j} = 0$ imply

$$\lambda_1(1 - \pi^l) - \lambda_3 - \underline{x}_h \lambda_4 = 0 \quad \text{and} \quad \lambda_1(1 - \pi^l) - \lambda_3 - \hat{x}_h \lambda_4 = 0, \quad (\text{E.3})$$

$$\lambda_2 + \underline{x}_l \lambda_5 = 0 \quad \text{and} \quad \lambda_2 + \hat{x}_l \lambda_5 = 0. \quad (\text{E.4})$$

Hence, reservation productivities have to coincide again, i.e. $\underline{x}_j = \hat{x}_j$. For simplicity will just use \underline{x}_j from now on. Define $\tilde{G}_j(\underline{x}_j) \equiv \int_{\underline{x}_j}^\infty \tilde{x} dG_j(\tilde{x})$ and $\Gamma_j \equiv \frac{1}{r_j} \int_{\underline{x}_j}^\infty (\tilde{x} - \underline{x}_j) dG_j(\tilde{x})$

and note their relationship $r_j \Gamma_j = \tilde{G}_j(\underline{x}_j) - \underline{x}_j(1 - G_j(\underline{x}_j))$ which will be used frequently in what follows. Next, I set $\frac{\partial \mathcal{H}}{\partial \theta_h} = 0$ and eliminate $\lambda_1(1 - \pi^l) - \lambda_3$ using (E.3) to get

$$-e^{-rt}C_h + \lambda_4 r_h \Gamma_h (1 - \eta) q_h(\theta_h) = 0, \quad (\text{E.5})$$

which solved for λ_4 implies

$$\lambda_4 = \frac{e^{-rt}C_h}{(1 - \eta)q_h(\theta_h)r_h\Gamma_h} \quad \text{and} \quad \dot{\lambda}_4 = -r\lambda_4. \quad (\text{E.6})$$

Inserting again in (E.5) gives

$$\begin{aligned} \lambda_1(1 - \pi^l) - \lambda_3 &= \frac{e^{-rt}C_h \underline{x}_h}{(1 - \eta)q_h(\theta_h)r_h\Gamma_h} \quad \text{and} \\ \dot{\lambda}_1(1 - \pi^l) - \dot{\lambda}_3 &= -r(\lambda_1(1 - \pi^l) - \lambda_3). \end{aligned} \quad (\text{E.7})$$

Proceeding analogously for θ_l implies

$$\lambda_2 = \frac{e^{-rt}C_l}{(1 - \eta)q_l(\theta_l)r_l\Gamma_l} \quad \text{and} \quad \dot{\lambda}_2 = -r\lambda_2, \quad (\text{E.8})$$

$$\lambda_5 = \frac{e^{-rt}C_l \underline{x}_l}{(1 - \eta)q_l(\theta_l)r_l\Gamma_l} \quad \text{and} \quad \dot{\lambda}_5 = -r\lambda_5. \quad (\text{E.9})$$

The optimality condition for y_h reads $e^{-rt} - \lambda_4(\pi_h^x + \pi_h^n) = -\dot{\lambda}_4$. I eliminate λ_4 and $\dot{\lambda}_4$ to get the optimal job creation condition for high-skilled jobs

$$(1 - \eta)q_h(\theta_h)\Gamma_h = C_h \quad \text{or} \quad (1 - \eta) \left(\frac{x_h^e - \hat{x}_h}{r_h} \right) = \frac{C_h}{q_h^f}. \quad (\text{E.10})$$

Similarly, transforming $\frac{\partial \mathcal{H}}{\partial y_l} = e^{-rt} - \lambda_5(\pi^x + \pi^n + \pi^h) = -\dot{\lambda}_5$ gives the optimal low-skill job creation condition

$$(1 - \eta)q_l(\theta_l)\Gamma_l = C_l \quad \text{or} \quad (1 - \eta) \left(\frac{x_l^e - \hat{x}_l}{r_l} \right) = \frac{C_l}{q_l^f}. \quad (\text{E.11})$$

Combine those two conditions with my expressions for the co-states to get

$$\lambda_1(1 - \pi^l) - \lambda_3 = \frac{e^{-rt}\underline{x}_h}{r_h}, \quad \lambda_2 = \frac{e^{-rt}\underline{x}_l}{r_l}, \quad \lambda_4 = \frac{e^{-rt}}{r_h}, \quad \lambda_5 = \frac{e^{-rt}}{r_l}. \quad (\text{E.12})$$

Compute $\frac{\partial \mathcal{H}}{\partial e_l} = -\dot{\lambda}_2$, eliminate all known co-states and transform to get

$$\underline{x}_l - h - \frac{\eta}{1 - \eta}C_l\theta_l + \pi^h\Gamma_h + \pi_l^n\Gamma_l + \frac{\lambda_1}{e^{-rt}}(1 - \pi^l)\pi^h = 0. \quad (\text{E.13})$$

Note that this equation implies that $\dot{\lambda}_1 = -r\lambda_1$ and consequently $\dot{\lambda}_3 = -r\lambda_3$. Next, I calculate $\frac{\partial \mathcal{H}}{\partial u_h} = -\dot{\lambda}_1 = r\lambda_1$ which gives

$$\lambda_1 \tilde{r} = -e^{-rt} [C_h \theta_h - C_l \theta_l] + e^{-rt} \theta_h q_h(\theta_h) \Gamma_h - e^{-rt} \theta_l q_l(\theta_l) \Gamma_l. \quad (\text{E.14})$$

Use the job creation conditions (E.10) and (E.11) to eliminate $q_j(\theta_j) \Gamma_j$ by $\frac{C_j}{1-\eta}$ and rearrange to arrive at

$$\frac{\lambda_1}{e^{-rt}} = \frac{\eta}{1-\eta} \left[\frac{C_h \theta_h - C_l \theta_l}{\tilde{r}} \right]. \quad (\text{E.15})$$

Insert this expression in (E.13) to derive the optimal job destruction condition for low-skilled workers

$$\hat{x}_l - h - \frac{\eta}{1-\eta} C_l \theta_l + \pi^h (1 - \pi^l) \frac{\eta}{1-\eta} \left[\frac{C_h \theta_h - C_l \theta_l}{\tilde{r}} \right] + \pi_l^n \Gamma_l + \pi^h \Gamma_h = 0. \quad (\text{E.16})$$

Compute $\frac{\partial \mathcal{H}}{\partial e_h} = -\dot{\lambda}_3$ and eliminate $-\dot{\lambda}_3$ using $\lambda_3 = e^{-rt} (1 - \pi^l) \frac{\eta}{1-\eta} \left[\frac{C_h \theta_h - C_l \theta_l}{\tilde{r}} \right]$. Rearranging reveals the optimal job destruction condition for high-skilled workers

$$\hat{x}_h - h - \frac{\eta}{1-\eta} \left[\frac{(1 - \pi^l) r}{\tilde{r}} C_h \theta_h + \frac{\pi^l (1 + r)}{\tilde{r}} C_l \theta_l \right] + \pi_h^n \Gamma_h = 0. \quad (\text{E.17})$$

Observe how $\pi^l = \pi^h = \pi^x = 0$ make the conditions collapse to their intragroup forms as derived in appendix section D.

F More comparative statics for the intragroup model

F.1 JD-JC diagram

Note that the determinant of the Jacobian of the JD-JC system is always positive, as $JD_\theta \equiv \frac{\partial JD}{\partial \theta} < 0$, $JD_{\hat{x}} \equiv \frac{\partial JD}{\partial \hat{x}} > 0$, $JC_\theta \equiv \frac{\partial JC}{\partial \theta} < 0$, and $JC_{\hat{x}} \equiv \frac{\partial JC}{\partial \hat{x}} < 0$, i.e. $\text{Det}(JDJC) = JD_\theta JC_{\hat{x}} - JC_\theta JD_{\hat{x}} \equiv \nabla > 0$. The elements of the inverse of the Jacobian of $JDJC$ system have the following signs

$$(\text{Jac}_{JDJC})^{-1} = \nabla^{-1} \begin{pmatrix} JC_{\hat{x}} & -JD_{\hat{x}} \\ -JC_\theta & JD_\theta \end{pmatrix} = \begin{pmatrix} - & - \\ + & - \end{pmatrix}.$$

To prove that the JD-curve slopes upward and the JC-curve is downward sloping proceed as follows. Total differentiation of the JD-curve w.r.t. \hat{x} and θ gives

$$\frac{(1 - \tau)(1 - G(\hat{x}))\pi^n + (1 - \tau)r}{\pi^n + r} d\hat{x} = \frac{\omega c}{1 - \omega} d\theta, \text{ hence}$$

$\frac{d\theta}{d\hat{x}}|_{\text{JD}} > 0$, the JD-curve is increasing.

Before deriving the slope of the JC-curve, let me define $\frac{\partial x^e}{\partial \underline{x}} = \frac{g(\underline{x})(x^e - \underline{x})}{1 - G(\underline{x})} \equiv \Psi$.

Assumption F.1. $\Psi < 1$. This is true in any case for some distributions (e.g. uniform, normal, ...) and very likely to be true for others (e.g. log-normal, with sufficiently small variance)³⁹.

Again, total differentiation reveals that

$$\left[\frac{(1 - \tau)(1 - \omega)}{\pi^n + r} (\Psi - 1) - \frac{c}{(q^f)^2} g(\underline{x})q(\theta) \right] d\hat{x} = \frac{\eta c}{q^w} d\theta,$$

$\frac{d\theta}{d\hat{x}}|_{\text{JC}} < 0$, the JC-curve is decreasing.

F.2 Policy effects

If total effects are not mentioned, it means that they are ambiguous.

Wage subsidy (D)

$$\frac{d\theta}{dD}|_{\text{JD}} = \frac{1 - \omega}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dD}|_{\text{JC}} = 0.$$

Effect: The JD-curve shifts outward. The JC-curve does not move. $\theta \uparrow, \hat{x} = \underline{x} \downarrow, u \downarrow$.

Hiring subsidy (H)

$$\frac{d\theta}{dH}|_{\text{JD}} = 0 \quad \text{and} \quad \frac{d\theta}{dH}|_{\text{JC}} = -\frac{q^w(1 - \omega)[\Psi - 1]}{\eta c} > 0.$$

Effect: The JD-curve does not move. The JC-curve shifts outward. $\theta \uparrow, \hat{x} \uparrow, \underline{x} < \hat{x}$. To determine the effect on the direction of \underline{x} see proposition 2.1.

Recruitment subsidy (R)

$$\frac{d\theta}{dR}|_{\text{JD}} = \frac{\theta}{c} > 0 \quad \text{and} \quad \frac{d\theta}{dR}|_{\text{JC}} = \frac{\theta}{\eta c} > 0.$$

Effect: The JD- and the JC-curves shift outward. $\theta \uparrow$. As the JC-curve moves stronger this implies that $\hat{x} = \underline{x} \uparrow$.

Firing tax (F)

³⁹It is easy to analytically show that for the uniform distribution $\Psi = 1/2, \forall \hat{x}$. Statements about the other distributions are based on numerical simulations.

$$\frac{d\theta}{dF}|_{\text{JD}} = \frac{(1-\omega)r}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dF}|_{\text{JC}} = \frac{q^w(1-\omega)[\Psi - 1]}{\eta c} < 0.$$

Effect: The JD-curve shifts outward and the JC-curve shifts inward. $\hat{x} \downarrow, \underline{x} > \hat{x}$. Using the implicit function theorem one can show that $\theta \downarrow$.

Output taxes (τ)

$$\frac{d\theta}{d\tau}|_{\text{JD}} = -\frac{(1-\omega)[(x^e - \hat{x})(1 - G(\hat{x}))\pi^n + (\pi^n + r)\hat{x}]}{\omega c(\pi^n + r)} < 0.$$

$$\frac{d\theta}{d\tau}|_{\text{JC}} = -\frac{q^w(1-\omega)[(x^e - \hat{x})(1 - \tau) - \Psi(F - H)(\pi^n + r)]}{(1-\tau)(\pi^n + r)\eta c}$$

This expression is smaller than 0, i.e. the JC shifts inward, whenever $F = H$. The bigger F in comparison to H , the smaller the inward shift.

Effect: The JD- and the JC-curves shift inward. $\theta \downarrow$.

Wage taxes (t)

$$\frac{d\theta}{dt}|_{\text{JD}} = -\frac{z(1-\omega)}{(1-t)^2\omega c} < 0 \quad \text{and} \quad \frac{d\theta}{dt}|_{\text{JC}} = 0.$$

Effect: The JD-curve shifts inward. The JC does not move, implying $\theta \downarrow, \hat{x} \uparrow$.

G Tables

Table G.1: Variable names

$j = l, h$	subscript indicating the skill type
$\hat{\cdot}$	hat notation refers to 'inside'-variables
\bar{x}	average productivity
\bar{w}	average wage
ω	bargaining weight for the worker
$\underline{\kappa}$	bound (lower) for $G(\cdot)$
$\bar{\kappa}$	bound (upper) for $G(\cdot)$
$G(x)$	cdf for productivity draws
X^e	conditional expectation of some random variable X w.r.t. \underline{x}
$X^{\hat{e}}$	conditional expectation of some random variable X w.r.t. $\hat{\underline{x}}$
∇	determinant of the JD-JC system
F	firing taxes
H	hiring subsidy
h	home production
z	instantaneous value of non-work ($z = b + h$)
r	interest rate
L	labor force
θ	labor market tightness
e	mass of employed people
u	mass of unemployed people
$\tilde{G}(x)$	partial expectation of productivity
$g(x)$	pdf for productivity draws
π^l	prob. of downgrade
π^x	prob. of exogenous separation
q^f	prob. of filling a vacancy
q^w	prob. of finding and accepting a job
$q(\theta)$	prob. of match for the firm
$\theta q(\theta)$	prob. of match for the worker
π^n	prob. of new productivity draw
π^h	prob. of upgrade
x	productivity, individual
R	recruitment subsidy
\underline{x}	reservation productivity, 'outside'
$\hat{\underline{x}}$	reservation productivity, 'inside'
Ω	social welfare
$S(x)$	surplus function
y	total production
τ	output tax rate
b	unemployment compensation
C	vacancy creation costs (gross)
c	vacancy creation costs (net of subsidies, i.e. $c = C - R$)
U	value of a being unemployed
$J(x)$	value of employment for the firm
$W(x)$	value of employment for the worker
V	value of a vacancy
D	wage subsidy (lump-sum)
t	wage tax rate
η	weight in the matching function

Table G.2: Overview calibration turbulence model

Variable	Target	Target value	Data source
r	-	0.0025	Eurostat, 1997-2007
π^h	-	0.0025	-
π_h^x	u	0.09	Eurostat, LFS, 2004-2014
π_l^x	u_l/u	0.38	Eurostat, LFS, 2004-2014
π^l	e_l/e	0.22	Eurostat, LFS, 2004-2014
π_j^n	$d \ln(\text{sep. rate})/d \ln \theta$	-0.21	EC (2013), average over 22 countries [range: -0.3 to -0.15]
C_h	θ_h	1	normalization
C_l	θ_l	1	normalization
A_h	u duration	14.78	OECD, duration in months, 2004-2014
A_l	$(u_h \text{ duration})/(u_l \text{ duration})$	0.52	Eurostat, LFS 2009 ad-hoc module
$\bar{\kappa}_h$	-	2	normalization
$\underline{\kappa}_h$	-	1	normalization
$\bar{\kappa}_l$	\bar{w}_h/\bar{w}_l	1.42	Eurostat, SES 2010
$\underline{\kappa}_l$	-	$\underline{\kappa}_h/1.42$	Eurostat, SES 2010
t_h	t_h/t_l	1.15	OECD tax-benefit calculator 2010 on SES 2010 earnings (weighted average)
t_l	budget constraint	-	-
b_l	$b_l/((1-t_l)\bar{w}_l)$	0.65	OECD tax-benefit calculator 2010 on SES 2010 (weighted average)
b_h	$b_h/((1-t_h)\bar{w}_h)$	0.55	OECD tax-benefit calculator 2010 on SES 2010 (weighted average)
h	$d \ln u/d(b/w)$	2.3	Costain and Reiter (2008) [range: 2 to 3]
η	$1 - d \ln(\text{find. rate})/d \ln \theta$	0.64	EC (2013), average over 22 countries
ω	Hosios condition	0.64	-

Note: LFS is short for Labour Force Survey, SES for Structure of Earnings Survey.