Cyclical unemployment benefits and non-constant returns to matching

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Abstract

Beside the function of insuring workers against the risk of unemployment, unemployment benefits might also be used as a Pigouvian instrument. When the magnitude of distorted aggregate search depends on the state of the economy along the business cycle, unemployment benefits optimally have to vary over the cycle too, even if workers are risk-neutral. The paper presents a search and matching model that explicitly allows for non-constant returns to matching which generate this type of externalities. It is shown that without policy intervention a generalized Hosios condition guaranteeing constrained efficiency cannot hold for any state of the economy. The implementation of the optimal allocation involves pro-cyclical (counter-cyclical) benefits correcting for excessive (too little) aggregate search activity if the matching function exhibits decreasing (increasing) returns to scale. Numerical simulations suggest that the cycle-dependent Pigouvian role of benefits is quantitatively rather limited compared to the classical function of insurance provision. This conclusion, however, is directly related to the Shimer-puzzle and the implausibly low responsiveness of the match surplus to productivity shocks for this class of models.

Keywords: search and matching, cyclical benefits, non-constant returns, efficiency

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1 Introduction

Ever since the seminal work by Baily (1978) the literature has studied questions of optimal unemployment benefits (UB) in frameworks that allow a trade-off between reduced incentives to search and gains from insurance. While many extensions and related issues[1] like optimal duration dependence (e.g. Hopenhayn and Nicolini, 1997 and Shimer and Werning, 2008) or liquidity concerns (Chetty, 2008) have been addressed, surprisingly the question of how to set UB optimally over the business cycle has only very recently sparked the interest of economists. Moyen and Stähler (2009) and Andersen and Svarer (2010) show that UB should be counter-cyclical if the government faces an *intertemporal* budget constraint in order to help workers to smooth consumption over the cycle. Many papers like Kiley (2003), Sánchez (2008), Kroft and Notowidigdo (2011) and Schmieder et al. (2011) emphasize a different argument. UB generosity should be counter-cyclical if the disincentive problem is less severe in downturns. Landais et al. (2010) provide a theoretical reason for such a mechanism. They also derive counter-cyclical optimal UB by arguing that the negative within-group search externality is stronger in bad times, i.e. the own search is more harmful to other searchers if the market is tight or jobs are rationed. This implies that from a social point of view, there is too much search in bad times and UB should be more generous in recessions. This search correcting function of UB will be refer to as the Pigouvian role. This additional congestion effect is not present in the canonical Diamond-Mortensen-Pissarides (DMP) model. Landais et al. (2010) introduce it by assuming decreasing marginal product of labor and sticky wages which generates 'job rationing'[2] and excessive search in bad times. While the principle economic intuition is very appealing this framework might be subject to the 'Pissarides (2009)-critique': Although a high degree of wage stickiness is observed during employment spells, 'outside' wages for new employees, which matter for job creation, vary almost one for one with productivity as given by standard Nash bargaining.

Without running afoul of the 'Pissarides-critique', this paper provides an alternative theoretical motivation for state dependent variation in the extend of search externalities. This is achieved by allowing for non-constant returns to scale in matching. Hence, in bad times the labor market does not only experience lower productivity but also more (or less) congestion than is socially optimal due to changes in employment *levels* depending on whether returns to scale are decreasing (or increasing). This change in congestion is symmetric for both workers and firms in contrast to the typical search externalities that are negative within the groups of workers or firms but positive between them. Keller

[1] See Fredriksson and Holmlund (2006) for a survey.
[2] See Michaillat (2012) for details.

et al. (2010) established that efficiency can be restored by an employment tax (subsidy) if returns are decreasing (increasing). This is shown by resorting to an abstract proportional match surplus tax/subsidy. I will show that such a tax/subsidy is not at the disposal of a policy maker as it cannot be mimicked by any proportional taxation or subsidization of economic flows[3]. Hence, any efficient policy has to be a function of the match surplus and therefore of the business cycle. I establish that UB have to be set pro-cyclically (counter-cyclically) if the matching function exhibits decreasing (increasing) returns to scale even if workers are risk-neutral which isolates the Pigouvian role of UB from their function of insurance provision.[4]

The existing literature almost exclusively used constant returns to matching.[5] Typical arguments brought up in favor of using constant returns to scale specifications are: analytic simplicity, consistency with a balanced growth path and empirical support. I will shortly comment on all these points. Using constant returns typically reduces the dimensionality of the matching process by one and makes equilibrium recursive and easily tractable. This is a fair argument and its validity obviously depends on the reduction in complexity one is willing to make and that seems to be appropriate for a given problem. The argument that only constant returns to scale are consistent with a balanced growth path cannot be supported unreservedly. The argument is certainly valid with respect to population or labor force growth that will lead to different growth rates of total output depending on the returns to scale assumption. However, non-constant returns to scale matching functions, as presented in this paper, are perfectly in line with a balanced growth path concerning exogenous technology growth. I will now turn to the question of empirical evidence. Probably the most comprehensive treatment[6] is the meta study by Petrongolo and Pissarides (2001) which is often used as a reference to justify constant returns to scale. Table 1.1 reproduces the unrestricted estimates for the matching elasticity of unemployment $(L - N)$ and vacancies (V) for the studies mentioned in Petrongolo and Pissarides (2001) that explicitly test for constant returns to scale. By inspection of this table it seems hard to claim that there is strong evidence for constant returns to scale of the matching function. Anderson and Burgess (2000) for the United States, Layard et al. (1991), Pissarides (1986) and Coles and Smith (1996) for the United Kingdom, and van

[3]Except for employment no other stocks are present in the model.

[4]Some authors like Marimon and Zilibotti (1999), Acemoglu and Shimer (2000) and Acemoglu (2001) stress an additional function of UB, also independent of the insurance provision argument. UB gives workers the time to look for more suitable jobs which increases productivity. As jobs are homogeneous in my model this function will not play a role.

[5]Notable exceptions are Diamond (1982), Pissarides (1984), Howitt and McAfee (1987) and Hyde (1997). Keller et al. (2010) provide a comprehensive summary of these papers.

[6]Another exhaustive summary of empirical results concerning the matching function is presented by Broersma and van Ours (1999).

Ours (1991) for the Netherlands support constant returns to scale. In contrast Warren (1996) and Blanchard and Diamond (1991) for the United States, Yashiv (2000) for Israel, Kangasharju et al. (2005) for Finland and Münich et al. (1999) for the Czech Republic find increasing returns. Decreasing returns are supported by Burda and Wyplosz (1994) for France, Germany, Spain and the United Kingdom, by Berman (1997) for Israel, by Burgess and Profit (2001) for the United Kingdom and by Fahr and Sunde (2004) for Germany.

Table 1.1: Empirical evidence on matching elasticities

	Sample	$\ln(L-N)$	$\ln(V)$	crts test
Pissarides (1986)	UK	-	-	✓
Layard et al. (1991)	UK	-	-	✓
Bergman (1997)	Israel	0.29	0.39	×
Burda and	France	0.52	0.09	×
Wyplosz (1993)	Germany	0.68	0.27	×
	Spain	0.12	0.14	×
	UK	0.67	0.22	×
Yashiv (2000)	Israel	0.49	0.87	×
Warren (1996)	USA	sum 1.33		×
Anderson and	USA: All new hires	0.43	0.81	✓
Burgees (2000)	USA: previously not employed	0.39	0.75	✓
	USA: previously employed	0.54	0.87	✓

Note: Reproduced from Pretongolo and Pissarides (2001). Only studies with tests for constant returns to scale (crts) are included. The tests are reported at a 10%-level. $L-N$ and V denote number of unemployed and vacancies.

Fahr and Sunde (2004) further estimate matching elasticities for different occupations and find substantial heterogeneity at this disaggregate level. Crafts and technical occupations seem to exhibit increasing returns while industrial, white collar and social occupations are related with decreasing returns. This result suggests that policy implications might vary considerably between groups of occupations. Broersma and van Ours (1999) and Sunde (2007) argue that estimates concerning the returns to scale can be severely biased by neglecting unobserved search intensity and on-the-job search. While former can be corrected relatively easy by making careful distinctions between 'conditional' and 'unconditional' matching functions[7] when relating empirical estimates to the theoretical model, correcting for the latter is less trivial and will not be attempted in this paper.

The aggregate matching function is usually used as a 'black box' tool. There have been several attempts of microfounding the matching function. Non of them suggests a regularity such that the underlying frictions can only result in a constant returns to scale specification. One of the first microfoundations is due to Butters (1977) and Hall (1979)

[7] I use the terminology as in Stevens (2007).

and reflects a coordination failure by mimicking the problem of randomly placing balls in urns. They derive the following matching function $\mathcal{M} = V\left[1-(1-1/V)^{L-N}\right]$ which has decreasing returns and only approximately converges to a constant returns to scale specification for large V, namely $\mathcal{M} = V(1-e^{-(L-N)/V})$. In an extension, Calvó-Armengol and Zenou (2005) microfound matching in a framework where workers are embedded in a social network and can find vacancies also through word-of-mouth. They show that the network size plays a crucial role for the efficiency of their matching process. On one hand, if the network size increases, the coordination failure shrinks because unemployed workers potentially learn about more job opportunities. On the other hand, coordination failure increases as it becomes more likely that a single worker receives several job offers at the same time. First, the positive effect dominates, then the negative effect is more important. Hence, for small networks the returns to scale are increasing while they are decreasing for oversaturated networks. Other microfoundations focus on mismatch of the form that workers and vacancies are randomly assigned to submarkets $\ell = 1, 2, \ldots$ that clear, i.e. $\mathcal{M}_\ell = \min\{L_\ell - N_\ell, V_\ell\}$, but where workers and vacancies are immobile and cannot (or only slowly) move to another submarket once they found themselves on the long side of their current submarket. This type of friction has been discussed in the literature since the 1970s, see for example Hansen (1970). More recent papers are provided by Lagos (2000) and Shimer (2005b). Latter derives an aggregate matching function that is increasing in market size, i.e. the measures of workers and vacancies in each submarket, although in a simulation exercise the matching function is virtually indistinguishable from a Cobb-Douglas specification. Stevens (2007) models the underlying friction as an explicit time consuming process of searching and evaluating potential matches, as they are heterogeneous, in form of a telephone-line-queuing model. She derives the following aggregate matching function[8] $\mathcal{M} = P_{accept} \cdot \frac{(L-N)V}{L-N+V}$, where P_{accept} is the probability that a job offer is accepted. Observe that the second term is homogeneous of degree one, but the matching function shares this property only if P_{accept} is independent of $L-N$ and V in levels. It is easy to generate a scenario where this is not the case, e.g. the existence of a simple welfare state where employed workers pay taxes and unemployed workers receive tax-financed benefits introduces level-dependence in job acceptance rates.

To summarize, first, there are no fundamental reasons derived from explicit microfoundation of why the matching function should exhibit constant returns to scale. Second, as the empirical evidence concerning the returns to scale of the matching process seems to be rather inconclusive and mixed with a high degree of country, regional and occupation specific heterogeneity, it seems reasonable to allow for a more flexible specification of the matching function in the theoretical model, as I will do in this paper.

[8] I normalized search costs to 1 without loss of generality for my argumentation.

The paper further relates to the strand of the literature that - in contrast to Chetty (2006a) - has focused on the insurance efficiency trade-off in *general equilibrium* settings where job finding probabilities are influenced by firms' behavior through vacancy creation and wage bargaining. Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001), Coles and Masters (2006), Lehmann and van der Linden (2007), and Coles (2008) look at optimal policy in steady state while Mitman and Rabinovich (2011) also take business cycle dynamics into account. Jung and Kuester (2011) extend their analysis by looking at a broader set of instruments and find that all, a recruitment subsidy, a firing tax and unemployment benefits have to rise in recession. However, none of those papers deals with the Pigouvian role of UB to correct for inefficient levels of aggregate search intensity as in this paper.

Probably the closest related study is the paper by Keller et al. (2010). They introduce a flexible matching function nesting different returns to scale specifications and derive the mentioned efficiency results which is also used in this paper. However, the main focus of their paper is not the implementation of an optimal allocation but rather market size effects introduced through non-constant returns to scale when labor force participation is endogenous. They further emphasize the different dynamic behavior of job finding rates in their model in contrast to the canonical DMP model where job finding probabilities directly jump to the new equilibrium values after a shock.

The paper is organized as follows. Section 2 extends the canonical DMP model such that it additionally allows for non-constant returns to scale in matching, decreasing returns to labor in production, and sticky wages. It can therefore nest different combinations of assumptions. Section 3 derives equilibrium while section 4 characterizes the social optimal allocation and possible implementations. Section 5 adds risk-aversion to analyze the interaction of the insurance and the externality correction purpose of UB. The section closes with a small numerical illustration. Section 6 concludes.

2 Model description

The model is set up in discrete time and I will focus exclusively on its comparative static behavior at the steady state.[9] All values are denoted at end of period. The model is

[9]As labor productivity is highly persistent and worker flows are large for the United States, for which I will calibrate the numerical example, looking at elasticities at the deterministic steady state is a legitimate approximation. I will comment on the issue of intertemporal insurance in case of risk-averse workers in section 5.

based on the simple, canonical DMP model as presented in Pissarides (2000).[10]

2.1 Matching technology

In contrast to the textbook model I allow for the following general form of the matching function similar to Keller et al. (2010). The number of total matches $\mathcal{M}$ is

$$\mathcal{M} = \Phi(m(s(L-N), V)) = \Phi(m(L \cdot su, L \cdot v)), \tag{2.1}$$

where L denotes exogenously fixed labor force, V is the number of vacancies, N is the number of employees, such that $L-N$ denotes the number of unemployed. u is the unemployment rate, v is the vacancy rate and s denotes average search effort. I interpret m as amount of aggregate search activity.[11] $m(\cdot,\cdot)$ is assumed to be increasing in both arguments and homogeneous of degree one. Because of that one can rewrite $\mathcal{M} = \Phi(Lu \cdot m(s,\theta))$, where θ denotes labor market tightness of form $\theta \equiv \frac{V}{L-N} = \frac{v}{u}$. The elasticity[12] of m w.r.t. u and v is denoted $\epsilon_u^m \equiv \eta$ and $\epsilon_v^m \equiv 1-\eta$, respectively, and assumed to be constant. The function $\Phi(x)$ for some $x > 0$ is strictly increasing and has a constant elasticity denoted by $\epsilon_x^\Phi \equiv \xi$. If $\xi = 1$ one is back in the canonical model with constant returns to scale. $\xi < (>)\ 1$ implies decreasing (increasing) returns. Consequently, the elasticities of $\mathcal{M}$ w.r.t. u and v are $\epsilon_u^{\mathcal{M}} = \eta \cdot \xi \equiv \eta_u$ and $\epsilon_v^{\mathcal{M}} = (1-\eta)\cdot\xi \equiv \eta_v$, respectively. Although, I allow for increasing returns to *scale* I rule out increasing returns to every single factor, i.e. $\eta_u, \eta_v \in (0,1)$.[13] Hence, $\mathcal{M}$ is concave in both u and v. The following further conditions have to be fulfilled by $\mathcal{M}$ by assumption: $\mathcal{M} \leq \min\{L-N, V\}$, $\mathcal{M} = 0$ if either $L = N$ or $V = 0$ and $L >> 1$ such that $L \cdot m(su, v) > 1$. Define the following

$$q^w(s_i, s, u, \theta) = s_i \frac{\mathcal{M}}{Lsu}, \tag{2.2}$$

as the matching probability for worker i where the matching probability per unit of search $\frac{M}{Lsu}$ is multiplied by the worker's individual search effort s_i. As workers are homogeneous they will always pick the same search effort in equilibrium, i.e. $s_i = s$, in which case every worker faces the following probability

$$q^w(s, u, \theta) = \frac{\Phi(Lu \cdot m(s,\theta))}{Lu}, \tag{2.3}$$

[10] I use almost the same notation. The bargaining power of the worker is denoted by ω and the probability of an exogenous split is π^x.

[11] 'Aggregate' refers to all workers and all firms.

[12] Throughout this paper elasticities of some variable y w.r.t. some variable x are denoted $\epsilon_x^y \equiv \frac{\partial y}{\partial x} \cdot \frac{x}{y}$.

[13] This is supported by empirical evidence, see Petrongolo and Pissarides (2001), and furthermore has the important theoretical implication that the negative within group congestion externalities are still present.

while the matching probability of a firm is simply $q^f(s,u,\theta) \equiv \frac{q^w(s,u,\theta)}{\theta}$. Note that in this general formulation q^w can also depend on the *level* of unemployment not only *relative* unemployment reflected by the labor market tightness like in the canonical model. Four important derived elasticities[14] that will be used extensively are

$$\epsilon_{s_i}^{q_i^w}|_{s_i=s} = 1 > 0, \quad \epsilon_s^{q^f} = \eta_u > 0, \quad \epsilon_\theta^{q^w} = \eta_v > 0, \quad \epsilon_u^{q^w} = \xi - 1. \tag{2.4}$$

Note that the sign of $\epsilon_u^{q^w}$ depends on our returns to scale assumption. If returns to scale are increasing, i.e. $\xi > 1$, then there is an additional positive level effect of the number of searches on the efficiency of matching. In a business cycle context this would imply that while the direct effect of lower factor productivity clearly increases unemployment the effect is dampened by the fact that for a higher number of searchers matching becomes more effective. On the other hand if $\xi < 1$ the direct effect is enforced as matching becomes less effective if u is high. The job finding rate reacts as follows to the degree of returns to scale and the market size

$$\epsilon_\xi^{q^w} = \xi \cdot \ln(L \cdot m(su,v)) > 0, \qquad \epsilon_L^{q^w} = \xi - 1. \tag{2.5}$$

Clearly, unless $\xi \neq 1$ the job finding rate is influenced by the market size. Absolute employment N evolves as follows

$$N' = \mathcal{M} + (1-\pi^x)N. \tag{2.6}$$

Next period's employment N' is simply the current stock minus separations plus the number of new matches. From a firm's perspective[15] this law of motion can be rewritten as

$$N' = q^f V + (1-\pi^x)N. \tag{2.7}$$

From the perspective of the workers the law is written as $N' = q^w \cdot (L-N) + (1-\pi^x)N$. Evaluating in steady state gives the standard Beveridge curve,

$$N = L(1-u) = \frac{q^w}{\pi^x + q^w}L, \qquad L - N = Lu = \frac{\pi^x}{\pi^x + q^w}L. \tag{2.8}$$

2.2 Representative firm

There is a single, competitive, representative firm with the following production function

$$Y = aF(N) \quad \text{with} \quad F(N) = N^\alpha, \quad 0 < \alpha \leq 1. \tag{2.9}$$

[14] See the appendix for the derivations.

[15] As the representative firm will take matching probabilities as given it has to maximize profits subject to (2.7) instead of (2.6). See section 2.2.

Hence, the production function nests constant as well as decreasing[16] marginal product of labor. The parameter a is interpreted as an exogenous productivity shift parameter that will capture the state of the economy along the business cycle and will later be used for comparative static exercises. Although production is handled by a single firm, the firm is assumed to be naive in the following respects to capture important characteristics of a competitive market with a large number of participants.

Assumption 2.1. ***Naive representative firm:*** *(a) The firm takes the matching probability q^f as given. (b) The firm assumes $\frac{\partial w}{\partial N} = 0$.*

Assumption 2.1(a) implies that the firm does not internalize the typical search externalities that are present in the canonical framework. Assumption 2.1(b) states that the firm does not realize that it can influence the wage (if possible) by deciding how many vacancies to post. This is not an issue if wages are exogenous, or if they are bargained but marginal returns to labor are constant, i.e. $\alpha = 1$. For the case of bargained wages and decreasing marginal returns to labor, wages depend on N via the marginal product. Stole and Zwiebel (1996) showed that in a setting where the firm treats every worker as being marginal during the bargaining it would strategically over-hire to compress wages.[17] I abstract from this strategic component by assuming the firm to be naive in that respect. The firm's discounted profits for a given number of workers N can be recursively written as

$$\Pi(N) = \max_V \left[aF(N) - wN - cV + \Pi(N')\right] \frac{1}{1+r}, \quad \text{s.t. (2.7)}. \tag{2.10}$$

Define $J \equiv \Pi'(N)$ as the marginal value of an additionally filled position for the firm. The optimality condition reads

$$-c + Jq^f = 0 \quad \Rightarrow \quad J = \frac{c\theta}{q^w}. \tag{2.11}$$

The envelope condition implies that

$$J = \left[aF'(N) - w - N\frac{\partial w}{\partial N} + J(1-\pi^x)\right] \frac{1}{1+r}. \tag{2.12}$$

By assumption 2.1(b) one can rewrite the envelope condition as

$$J = \frac{y-w}{r+\pi^x}, \tag{2.13}$$

[16] A typical justification is that the capital stock is fixed or adjusts only sluggishly in the short run. Hence, labor reallocates faster than capital.

[17] Cahuc and Wasmer (2001) show that this result disappears if the production function has decreasing returns in both, labor and capital, but constant returns to scale.

where I defined output of a marginal worker as $y \equiv aF'(N) = aF'(L(1-u))$.[18] Combining the optimality and the envelope condition gives the typical job creation (JC) condition that equates benefits and costs of marginally increasing N, i.e.

$$\frac{y - w}{r + \pi^x} = \frac{c\theta}{q^w}. \tag{2.14}$$

2.3 Workers

Workers are assumed to be risk-neutral such that one can disentangle the welfare effect of UB because of 'search correction' in contrast to insurance.[19] A worker i can be in two discrete states: employed or unemployed. The corresponding values are denoted W_i and U_i,

$$U_i = \max_{s_i} \left[z_i + q_i^w W_i + (1 - q_i^w) U_i \right] \frac{1}{1 + r}, \tag{2.15}$$

where the instantaneous value of unemployment $z_i \equiv h + b - k(s_i)$ is home production plus UB minus search effort in monetary terms. The effort function has the usual properties, i.e. $k'(\cdot) > 0$ and $k''(\cdot) > 0$. The value of being employed is given by

$$W_i = \left[w_i - T + \pi^x U_i + (1 - \pi^x) W_i \right] \frac{1}{1 + r}. \tag{2.16}$$

Here T denotes a tax on employed workers which will finance UB for the unemployed. Maximization over search effort while taking all market variables as given implies the following first-order condition for optimal search

$$\frac{\partial q_i^w}{\partial s_i} (W_i - U_i) = k'(s_i), \tag{2.17}$$

which equates marginal benefits and costs of an additional unit of individual search effort. As mentioned before all workers are identical which implies that in equilibrium all workers choose the same search intensity, hence

$$W - U = \frac{s k'(s)}{q^w}. \tag{2.18}$$

Combine (2.15), (2.16) and (2.18) to get

$$W - U = \frac{w - T - z - s k'(s)}{r + \pi^x}. \tag{2.19}$$

[18] Clearly, in the case of constant returns to labor, $\alpha = 1$, marginal output is equal to the exogenous productivity parameter, i.e. $y = a$.

[19] This assumption is relaxed in section 5.

Inserting this expression in (2.18) gives the typical job search (JS) condition which reveals that search effort increases with the difference of wage and unemployment income

$$\frac{w - T - z - sk'(s)}{r + \pi^x} = \frac{sk'(s)}{q^w}. \tag{2.20}$$

2.4 Wage determination

I consider two wage determination rules. First, like in Landais et al. (2010) I allow for simple sticky wages of form

$$\textbf{\textit{sticky wage:}} \qquad w = w_0 a^\gamma, \quad 0 < \gamma < 1. \tag{2.21}$$

In this case the elasticity of the wage w.r.t. productivity, $\epsilon_a^w = \gamma$, is less than 1. Second, I allow for bargained wages subject to the following sharing rule[20]

$$W - U = \frac{\omega}{1-\omega} J. \tag{2.22}$$

Inserting (2.13) and (2.19) in (2.22) and solving gives the following wage schedule

$$\textbf{\textit{flexible wage:}} \qquad w = (1-\omega)\left[z + T + sk'(s)\right] + \omega y, \tag{2.23}$$

with an elasticity of the wage w.r.t. productivity of $\epsilon_a^w \approx 0.98$ for a reasonable calibration[21].

2.5 Surplus taxation

This short section explains how a proportional surplus tax t is introduced to the model for the regime of flexible wages. This tax is admittedly rather abstract but it will help in understanding the principle role optimal policy will have to play. Observe that in the case of no such tax the surplus is $S \equiv W - U + J$. Simply add (2.13) to (2.19) to arrive at[22]

$$S = \left[\frac{y - T - z - sk'(s)}{r + \pi^x}\right]. \tag{2.24}$$

Now a surplus tax is introduced such that only $(1-t)S$ is shared among the parties, i.e. $(1-t)S \equiv W - U + J$. Hence, although the sharing rule (2.22) is still valid the values J

[20] For risk-neutral workers this coincides with the first-order condition of a typical Nash bargaining problem, i.e. $w = \text{argmax}(W-U)^\omega J^{1-\omega}$.

[21] This value is stated in Pissarides (2009) and confirmed in the numerical section of this paper.

[22] Note that if one assumed that $k(\cdot)$ was of simply iso-elastic form with constant elasticity $\epsilon_s^k = \nu > 1$ one could rewrite the term $k(s) - sk'(s)$ as $(1-\nu)k(s)$. Consequently, $z + sk'(s) = b + h + (\nu - 1)k(s)$.

and $W - U$ are now given by

$$J = (1-\omega)(1-t)S \equiv \omega^f S, \qquad W - U = \omega(1-t)S \equiv \omega^w S, \tag{2.25}$$

where ω^f and ω^w are the *effective* bargaining powers for firm and worker. Hence, optimal job creation and optimal job search effort are characterized by the following two conditions

$$\omega^f S = \frac{c\theta}{q^w}, \qquad \omega^w S = \frac{sk'(s)}{q^w}. \tag{2.26}$$

The next section will describe this in more detail.

3 Equilibrium

In addition to the derived conditions also the government's budget has to be balanced, therefore the following has to hold

$$NT = (L - N)b \quad \Leftrightarrow \quad T = \frac{u}{1-u}b. \tag{3.1}$$

One can now simply eliminate T in all corresponding equations. Further, observe that I excluded the surplus tax from the government's budget, i.e. I assume it to be uncompensated. This has the following reason. In section 4 I argue that such a tax is not at the disposal of the policy maker and cannot be mimicked by a set of instruments (even if they are uncompensated as well). The tax will just help to characterize optimality. For the implementation of an optimal allocation it will not play a role. Equilibrium $\langle \theta^*, s^*, w^*, u^* \rangle$ is given by the simultaneous solution to the job creation condition (2.14), the optimal search condition (2.20), either one of the two wage conditions (2.21) or (2.23), and the Beveridge curve (2.8). Observe that one can reduce the system further by eliminating the wage. Hence, equilibrium is given by the vector $\langle \theta^*, s^*, u^* \rangle$ that solves

(a) for sticky wages:

$$\text{JC-wage}: \qquad \frac{y - w_0 a^\gamma}{r + \pi^x} = \frac{c\theta}{q^w(s,u,\theta)}, \tag{3.2}$$

$$\text{JS-wage}: \qquad \frac{w_0 a^\gamma - h - b/(1-u) + k(s) - sk'(s)}{r + \pi^x} = \frac{sk'(s)}{q^w(s,u,\theta)}, \tag{3.3}$$

(b) for flexible wages:

$$\text{JC-wage :} \qquad \omega^f \left[\frac{y - h - b/(1-u) + k(s) - sk'(s)}{r + \pi^x} \right] = \frac{c\theta}{q^w(s,u,\theta)}, \tag{3.4}$$

$$\text{JS-wage :} \qquad \omega^w \left[\frac{y - h - b/(1-u) + k(s) - sk'(s)}{r + \pi^x} \right] = \frac{sk'(s)}{q^w(s,u,\theta)}, \tag{3.5}$$

and for both:

$$\text{BC :} \qquad \frac{\pi^x}{\pi^x + q^w(s,u,\theta)} = u. \tag{3.6}$$

If one combines both optimality conditions from the flexible wage regime one arrives at a convenient alternative condition for (3.5)

$$sk'(s) = \frac{\omega}{1-\omega} c\theta. \tag{3.7}$$

This can alternatively be derived by combining (2.18), (2.22) and (2.11)

$$sk'(s) = q^w(W-U) = \frac{\omega}{1-\omega} q^w J = \frac{\omega}{1-\omega} c\theta. \tag{3.8}$$

The alternative condition (3.7) also allows us to write the wage equation in a slightly different way

$$w = (1-\omega)\left[z + T\right] + \omega(y + c\theta), \tag{3.9}$$

which is well-known from Pissarides (2000).

4 Social optimum and implementation by benefits

This section derives the optimal allocation a social planner would choose.[23] I will then discuss possible decentralizations of the allocation. As workers are risk-neutral, welfare and output maximization coincide[24]. The social planner faces the following recursive

[23] In the canonical model, an equilibrium is unique if it exists. Sufficient conditions are constant returns to scale in the matching function and linear search technology, see Pissarides (2000). Only the second condition is fulfilled in the present model. Diamond (1982) and Diamond (1984) established that increasing returns in the matching function could generate multiple equilibria. Pissarides (1986) analyzes uniqueness and multiplicity of equilibria in a more closely related model. It is important to realize that a social planner subject to the same matching technology faces the same indeterminacy concerning the allocation compared to the decentralized case. Obviously, he will pick the most efficient equilibrium. For the theoretical model I assume that agents can coordinate to do the same, i.e. I always focus on the most efficient equilibrium. Appendix C establishes conditions for multiplicity and uniqueness. I also find that increasing returns are necessary. However, in the numerical part multiplicity of equilibria was not an issue.

[24] Simply insert the resource constraint $aF(N) - cV = Nw + \Pi$, where Π denotes aggregate profits, to derive the utilitarian per-period welfare measure $Nw + (L-N)\left[h - k(s)\right] + \Pi$.

problem

$$\Omega(N) = \max_{V,s} \left[aF(N) + (L - N)\left[h - k(s)\right] - cV + \Omega(N')\right] \frac{1}{1+r}, \tag{4.1}$$

subject to (2.6). Note that the social planner in contrast to the agents does not take matching probabilities as given.

Let $S^S \equiv \Omega'(N)$ be the social value of filling an additional job, i.e. the social match surplus. Then the optimality conditions for V and s and the envelope condition for N evaluated in steady state are

$$V: \qquad -c + S^S \cdot \frac{\partial \mathcal{M}}{\partial V} = 0 \tag{4.2}$$

$$s: \qquad -k'(s)(L - N) + S^S \cdot \frac{\partial \mathcal{M}}{\partial s} = 0 \tag{4.3}$$

$$N: \qquad (1+r)S^S = aF'(N) - [h - k(s)] + S^S \cdot \frac{\partial \mathcal{M}}{\partial N} + (1 - \pi^x)S^S. \tag{4.4}$$

I use the following derivations

$$\frac{\partial \mathcal{M}}{\partial V} = \xi(1-\eta)q^f, \quad \frac{\partial \mathcal{M}}{\partial N} = -\xi\eta q^w, \quad \frac{\partial \mathcal{M}}{\partial s} = \xi\eta\frac{\mathcal{M}}{s}, \tag{4.5}$$

which have to be inserted in (4.2) to (4.4). The first condition states that vacancies should be created up to the point where the share $\eta_v = (1-\eta)\xi$ of the social value of the job equals the expected marginal costs of vacancy creation. The second condition implies that search is optimal if a share $\eta_u = \eta\xi$ of the social value of a job equals the workers' expected marginal search costs. Both can be written as

$$\eta_v S^S = \frac{c\theta}{q^w}, \qquad \eta_u S^S = \frac{sk'(s)}{q^w}. \tag{4.6}$$

Combine both conditions to get the following simple relation

$$sk'(s) = \frac{\eta}{1-\eta}c\theta. \tag{4.7}$$

Now combine the third and the second condition to get the social value of a job

$$S^S = \frac{y - [h - k(s)] - sk'(s)}{r + \pi^x}. \tag{4.8}$$

The social surplus is almost identical to the decentralized match surplus. The only difference is that UB can drive a wedge between them, i.e. $S^S = S + \frac{b/(1-u)}{r+\pi^x}$. Consequently, the two social optimality conditions for vacancy creation and search intensity read

$$\eta_v \left[\frac{y - [h - k(s)] - sk'(s)}{r + \pi^x} \right] = \frac{c\theta}{q^w}, \tag{4.9}$$

$$\eta_u \left[\frac{y - [h - k(s)] - sk'(s)}{r + \pi^x} \right] = \frac{sk'(s)}{q^w}. \tag{4.10}$$

I will now show how these optimality requirements relate to the decentralized equilibrium conditions first in a sticky wage environment and then more extensively in the flexible wage regime.

4.1 Sticky wages and job rationing

The model of Landais et al. (2010) is nested in the specification above for $\xi = 1$, $\alpha < 1$ and using (2.21) as wage equation. They find that even if workers are risk-neutral there is a 'job rationing' externality implying too much search that should be corrected by a positive UB level. The idea of 'job rationing' can be easily explained within the framework of the presented model. I will focus on the job creation decision and restate the corresponding condition (3.2)

$$\frac{aF'(N) - w_0 a^\gamma}{r + \pi^x} = \frac{c}{q^f}.$$

It is useful to look at the limiting case $c \to 0$ such that posting vacancies is costless and search frictions vanish in the limit. In the canonical model with constant marginal productivity, i.e. $\alpha = 1$, firms would post vacancies ad infinitum such that $\theta \to \infty$ and matching from the workers' perspective would occur instantaneously, i.e. $u \to 0$ and $N \to L$. In the model of Landais et al. (2010) this is not the case as jobs might be rationed at some point and no new jobs are created because the marginal product falls below the rigid wage.[25] Clearly, rationing is more likely to occur if factor productivity a is small, i.e. in a recession. It is easy to see that any additional uncoordinated search effort by the workers would be socially wasteful in such a situation.

In addition to decreasing returns to labor and wage stickiness there is another important deviation from the standard model in terms of the welfare measure. Landais et al. (2010) optimize workers' welfare (or workers' expected income in the case of risk-neutrality) but ignore aggregate profits for the derivation of the optimal UB formula in their stylized model[26]. I will show that their results heavily depend on this last assumption. Hence, it

[25]Actually, even with bargained, flexible wages jobs could be rationed as wages cannot fall below the workers per-period value of unemployment, while the marginal product can. Still, there is an important difference as this would be socially efficient rationing because a social planner would never want to maintain jobs that are less valuable than the value of home production.

[26]For the numerical simulation of their complex, dynamic model they assume that profits can be taxed

should come at no surprise that they find that benefits should always be positive as the positive intergroup externality of workers' search effort on aggregate profits is ignored. In contrast, I assume that the social planner takes profits into account and maximizes total output. Observe that the allocation is efficient if and only if, by chance, the following conditions holds

$$1-\eta = \frac{y - w_0 a^{\gamma}}{\Gamma^S} \quad \text{and} \quad b = 0, \quad \Gamma^S \equiv [y - (h - k(s)) - sk'(s)]\,. \tag{4.11}$$

Γ^S denotes the social per-period surplus, i.e. $S^S = \frac{\Gamma^S}{r+\pi^x}$. Equation (4.11) is derived by comparing (3.2) and (3.3) with (4.9) and (4.10). It basically states the classical Hosios-condition, namely that the share of the surplus claimed by the firm is equal to the elasticity of the matching function w.r.t. v. Clearly, the conditions stated in (4.11) differ from those derived by maximizing only the workers' expected income and imply that for any values of a and γ in principle there could be too much or too little unemployment. Further, in case of failure of the first condition, UB cannot be used to restore efficiency.

The assumption of wage stickiness has been a controversially discussed topic in the macro-labor literature and while conventional wisdom on wage rigidity is mostly based on time-series analysis working with aggregate wage levels this view has been recently challenged by studies looking at individual worker data that allow for an explicit distinction between wages of new and continuing jobs. Most prominently Pissarides (2009) showed that the wage rigidity of continuing jobs is irrelevant for the job creation decision of firms as firms just care about their share of the expected surplus and not about how exactly the surplus is split in future periods. Indeed, individual-worker studies, like Haefke et al. (2008), estimate the elasticity of wages for continuing jobs in the range $\epsilon_a^{w^c} \in [0.3, 0.5]$ but when they estimate the same elasticity for newly created jobs they find that those move much stronger with the cycle. Combining their point estimate with a more indirect estimate by Pissarides (2009) himself gives a range of $\epsilon_a^{w^n} \in [0.9, 1.02]$. Recall that applying the typical bargaining sharing rule results in $\epsilon_a^w \approx 0.98$ which is perfectly in line with the corresponding estimates. As wage stickiness during an employment spell does not affect the job creation condition, applying the bargaining sharing rule for all jobs should be preferred over using sticky wage specifications. This basically summarizes the argument of Pissarides (2009) which I will refer to as the 'Pissarides-critique' concerning wage stickiness. Gertler and Trigari (2009) and Blanchard and Galí (2010) responded to the 'Pissarides-critique' by arguing that the true elasticity of wages for new jobs w.r.t. productivity is hard to identify as the high estimates could in principle also stem from

and redistributed. However, it is hard to isolate the exact role of the search correcting effect of the benefits as the simulations are just carry out for the case of risk-aversion.

compositional effects as Haefke et al. (2008) did not control for occupational changes[27]. Nevertheless, to my knowledge no empirical study has convincingly confirmed real wage rigidity for newly created jobs which is why I will focus on the flexible wage specification as recommended by Pissarides (2009).

4.2 Flexible wages and non-constant returns to matching

Let me return to the optimality conditions (4.9) and (4.10). Compare this expressions to the decentralized conditions for flexible wages (3.4) and (3.5). Both condition pairs differ w.r.t. two attributes: the *size* of the surplus and *shares* of the surplus the firm and the worker receive. The social and the decentralized surplus just differ by a 'wedge' created if $b \neq 0$, i.e. $S^S = S + \frac{b/(1-u)}{r+\pi^x}$. The socially optimal shares are influenced by the degree of returns to scale ξ, while the decentralized shares depend on the surplus tax t. Take $t = b = 0$ for the moment. It is obvious that in case of constant returns, i.e. $\xi = 1$, the classical Hosios (1990)-condition $\omega = \eta = \eta_u$ implies efficiency. Now let us assume $\xi \neq 1$. The social planner takes congestion effects on the probability of finding a job through changes in the *level* of unemployment into account, which is not done by agents in the decentralized economy who naively treat q^w and q^f as given. As the classical Hosios-condition for constant returns to scale is given by $\omega = \eta$, this parameter constellation will serve as my benchmark in order to isolate the additional inefficiencies generated by $\xi \neq 1$. Similar to Keller et al. (2010) a generalization of the Hosios condition can be formulated as follows.

Proposition 4.1. ***Generalized Hosios-condition***: *In a regime with flexible wages the allocation is efficient if the effective bargaining powers and the total matching elasticities coincide, i.e. $\omega^w = \eta_u$ and $\omega^f = \eta_v$, and no other policy is in place. Given that the classical Hosios-condition, i.e. $\omega = \eta$, is fulfilled, the efficient allocation can be reached by setting a surplus tax/subsidy $t = 1 - \xi$ and $b = 0$.*

Proof. This follows directly from comparing (3.4) and (3.5) with (4.9) and (4.10). Note that a failure of $\omega \neq \eta$ cannot be corrected by adjusting t as efficiency along the job creation margin would require $t = 1 - \xi\frac{1-\eta}{1-\omega}$ while optimality along the job search margin demands $t = 1 - \xi\frac{\eta}{\omega}$.[28] ∎

Observe that the generalized Hosios-condition collapses to its classical form if $\xi = 1$. Given that $\omega = \eta$ and in absence of any policy instrument one can distinguish three cases. If $\xi = 1$ (constant returns) the level of employment is efficient. If $\xi < 1$ (decreasing returns)

[27]The argument is that in bad times an architect might be forced to take a low paying job as a cab driver, while if he found a job in an architectural firm he would have suffered a much smaller than almost proportional loss due to the fall in aggregate productivity.

[28]See for example Schuster (2010) for a discussion on how to correct in the case of $\omega \neq \eta$.

the level of employment is inefficiently high. If $\xi > 1$ (increasing returns) the level of employment is inefficiently low. This implies that with decreasing returns one requires an implicit employment tax to prevent excessive aggregate search, while in case of increasing returns one requires an implicit employment subsidy to foster aggregate search. How should the generalized Hosios-condition be interpreted? Recall that the classical Hosios-condition finds the optimal trade-off between the positive intergroup and the negative intragroup externalities. This is done by splitting the match surplus in an optimal way. If $\omega > \eta$ then workers would search too much while firms would exert too little search effort. The generalized version of the Hosios-condition in addition gives a formula for optimal *aggregate* search. If $\xi < 1$ then both, workers and firms, would provide too much search effort from a social perspective. That is why the surplus itself has to shrink in order to reduce search incentives for all agents. Importantly, this can only be achieved through policy intervention while a constant returns to scale setting in principle[29] allows equilibrium to be efficient without any intervention. Proposition 4.1 presents results for an employment tax/subsidy in form of a proportional surplus tax/subsidy as originally derived by Keller et al. (2010). The idea of a direct surplus tax is admittedly very abstract. This becomes even more clear with the following proposition.

Proposition 4.2. ***Impossibility of proportional surplus taxation****: There is no combination of proportional taxes on any economic activity flows that could mimic surplus taxation of the form (2.26).*

The proof is provided in the appendix. I imposed the reasonable assumption that a government cannot directly tax the match surplus[30]. Even if it could tax any economic activity flow like output, wages, benefits, vacancy posting costs, search costs or home production, proposition 4.2 states that *proportional* surplus taxation cannot be implemented. The intuition is that one would require proportional subsidization of vacancy posting or marginal search costs on one hand to tax the surplus proportionally while on the other hand one must not allow for such instruments to prevent distortion of the marginal cost components on the right hand side of (2.26).[31]

I have argued that the implementation of an efficient allocation using a proportional surplus tax/subsidy is impossible as a typical policy maker usually does not have an instrument like that at his disposal and it cannot be mimicked by other instruments. Instead of changing the *splitting weights* of the surplus to achieve proportional surplus

[29]As the parameters of the classical Hosios-condition, ω and η, are systematically unrelated, efficiency can only happen by chance and is unlikely to occur.

[30]The government still has to observe the surplus because optimal benefits will depend on it.

[31]Note that this result is directly related to the existence of a balanced growth path. Let output, the value of leisure and all other costs grow at a constant rate g. The factor $1 + g$ would simply cancel out in the first-order conditions, leaving the unemployment rate unchanged.

taxation/subsidization one could directly change the *size* of the surplus by controlling the wedge between the social and the decentralized surplus. In this framework this can be done by setting UB accordingly.[32] I still assume $\omega = \eta$ to isolate the additional from the conventional search externalities. Set $t = 0$ and subtract (4.9) from (3.4) and solve for the optimal UB level. Equation (4.12) gives the optimal size of UB in absence of any insurance motive.[33]

$$b = \Gamma^S(1 - \xi)(1 - u). \tag{4.12}$$

Observe that this is indeed the optimal level of b as it also implies optimal search intensity. If $\xi = 1$ the optimal level of benefits is $b = 0$. If $\xi > 1$ it is $b < 0$[34] which implies that otherwise there would be too little search. In case of decreasing returns, $\xi < 1$, there is excessive search which requires $b > 0$. Another important implication of (4.12) is that for $\xi \neq 1$ the optimal b has to move proportionally with the surplus itself. The results are summarized in the following proposition.

Proposition 4.3. ***Optimal unemployment benefits***: *The allocation is efficient if unemployment benefits are set according to (4.12). In case of decreasing (increasing) returns to scale optimal UB are positive (negative) and rise (fall) with productivity.*

The proof is provided in the appendix and simply relies on the trivial property that both the employment rate $1 - u$ and the social per-period surplus Γ^S are pro-cyclical. The different cases are illustrated in figure 4.1.

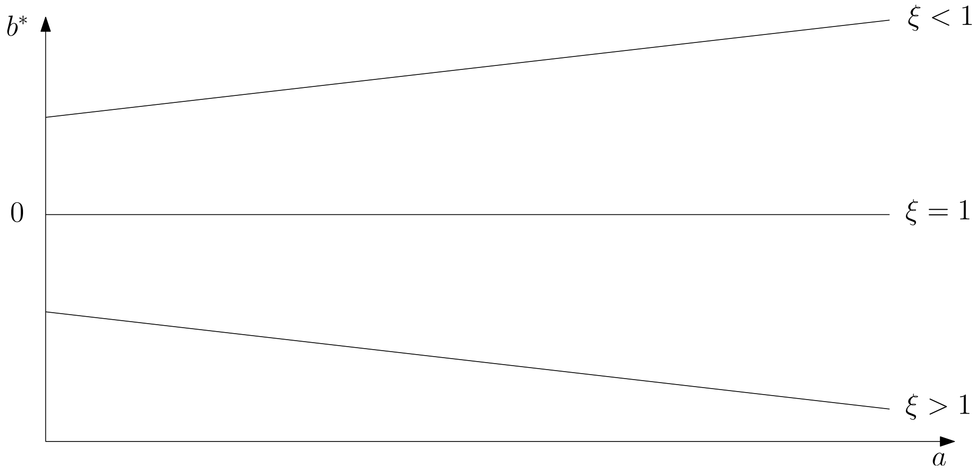
To summarize, if $\xi < 1$ then $b > 0$ and $db/da > 0$ i.e. benefits have to be set pro-cyclically. On the other hand if $\xi > 1$ then $b < 0$ and $db/da < 0$, i.e. benefits should be counter-cyclical. The finding of Landais et al. (2010) that benefits should be positive and counter-cyclical are a mixture of the results from both returns to scale scenarios. With decreasing returns to scale there is too much aggregate search, hence UB should be positive like in Landais et al. (2010), but in contrast benefits should *rise* in good times as unemployment shrinks which reduces the effectiveness of the matching process. If returns are increasing, the opposite is true and UB should be counter-cyclical because matching

[32] In principle any alternative tax that changes the surplus, e.g. an uncompensated output or profit tax, etc. could be used. However, if they have to compensated by a per-worker tax T they leave the surplus unchanged as they would just redistribute between the inside options and not between inside and outside options, as it is the case for per-worker tax-financed UB.

[33] In the present framework UB are adjusted directly. Appendix section D derives the optimal formula for the replacement rate.

[34] Here I assume that the policy maker can set $b < 0$ i.e. he can tax home production, which is typically not a feasible option. However, recall that the set-up with risk-neutral workers was done to simplify the analysis and isolate the Pigouvian role of UB. In a richer model with risk-aversion and positive unemployment insurance increasing returns to scale would rather imply a reduction in existing UB instead of negative UB, which is obviously feasible.

Figure 4.1: Optimal unemployment benefits over the business cycle for different degrees of returns to scale



is less effective if the level of unemployment is high, as in Landais et al. (2010), while too little aggregate search implies a negative optimal *level* of UB.

5 A numerical example and risk-aversion

This section serves to get an impression of how big the search correction actually has to be and how this quantitatively relates to the typical insurance provision function. I will therefore extend the existing model to allow for non-linear utility. Risk-aversion is introduced by wrapping per-period consumption flows into a felicity function $u(\cdot)$ with the properties $u'(\cdot) > 0$ and $u''(\cdot) < 0$.[35] I assume that utility from consumption and disutility from search effort are linearly separable. One can write the recursive values of being unemployed and employed as

$$U_i = \max_{s_i} \left[u(h+b) - k(s_i) + q_i^w W_i + (1-q_i^w)U_i\right] \frac{1}{1+r}, \tag{5.1}$$

$$W_i = \left[u(w_i - T) + \pi^x U_i + (1-\pi^x)W_i\right] \frac{1}{1+r}. \tag{5.2}$$

It is easy to see that the optimal job search condition is given by

$$\frac{u(w-T) - u(b+h) + k(s) - sk'(s)}{r+\pi^x} = \frac{sk'(s)}{q^w}. \tag{5.3}$$

[35]Note that because of convention I use the notation $u(\cdot)$ for the instantaneous utility function despite the fact that u also denotes the unemployment rate. The distinction becomes clear from the context.

For the determination of the wage I will again resort to a simple surplus sharing rule.

$$\check{W} - \check{U} = \frac{\omega}{1-\omega} J. \tag{5.4}$$

Here $\check{W}$ and $\check{U}$ denote the values of working and not working in pure monetary terms. I assume that utility is non-transferable, i.e. the two parties can only share the surplus evaluated in monetary terms. See Michau (2011) for more details on this surplus splitting rule.[36] Consequently, the wage equation does not change in comparison to the risk-neutral case and is again given by (2.23). Therefore also the condition for optimal vacancy posting is unchanged. Equilibrium is given by the vector $\langle \theta^*, s^*, w^*, u^*, T^* \rangle$ that solves the following system of equations

$$\text{JC}: \quad \frac{y-w}{r+\pi^x} = \frac{c\theta}{q^w}, \tag{5.5}$$

$$\text{JS}: \quad \frac{u(w-T) - u(b+h) + k(s) - sk'(s)}{r+\pi^x} = \frac{sk'(s)}{q^w}, \tag{5.6}$$

$$\text{wage}: \quad (1-\omega)\left[z + T - sk'(s)\right] + \omega y = w, \tag{5.7}$$

$$\text{government}: \quad \frac{u}{1-u} b = T, \tag{5.8}$$

$$\text{BC}: \quad \frac{\pi^x}{\pi^x + q^w} = u. \tag{5.9}$$

I now characterize the optimal allocation. To simplify the analysis I use $\alpha = 1$ such that the representative firm does not make profits. As an implementation of the first best allocation requires full insurance which is incompatible with bargained wages with positive bargaining power for the workers I will consequently look for a second best solution. In this case the planner maximizes utilitarian welfare subject not only to the law of employment but also to the implementability constraints given by the decentralized equilibrium conditions. For reasons of analytic convenience I will write the corresponding Bellman equation as a function of the unemployment rate instead of total employment. The second best optimization problem in recursive form looks as follows

$$\Theta(u) = \max_{\theta,s,b,w,T} \left[L(1-u)\cdot u(w-T) + Lu \cdot [u(h+b) - k(s)] + \Theta(u')\right] \frac{1}{1+r},$$
$$\text{subject to} \quad u' = (1-q^w)u + \pi^x(1-u), \tag{5.10}$$

and the system of equilibrium conditions (5.5) to (5.8). As the economic interpretability of the resulting first-order conditions is rather limited I will illustrate the case of risk-aversion using a small numerical example.

[36] A similar result can be derived when using a first-order Taylor approximation of the first-order condition of an explicit Nash bargaining game.

5.1 Calibration

First, the following functional forms for the utility function and the search effort function where chosen

$$m(su, v) = (su)^{\eta} v^{1-\eta}, \tag{5.11}$$

$$\Phi(x) = \mathcal{M}_0 x^{\xi}, \tag{5.12}$$

$$k(x) = k_0 x^{\nu}, \quad \nu > 1, \tag{5.13}$$

$$u(x) = \begin{cases} \frac{x^{1-\sigma}-1}{1-\sigma} + 1 & \text{if } \sigma \geq 0 \text{ and } \sigma \neq 1, \\ \ln(x) + 1 & \text{if } \sigma = 1. \end{cases} \tag{5.14}$$

Observe that $\sigma = 0$ implies $u(x) = x$ as used in the previous sections. The calibration mainly relies on the values chosen in Pissarides (2009). The benchmark calibration is done for a 'typical' separated labor market in the United States at quarterly frequency. Table 5.1 summarizes the choices for the parameters for the benchmark case. Worker flows, labor market tightness and consequently unemployment are taken from the Job Openings and Labor Turnover Survey (JOLTS) and the Help-Wanted Index (HWI) as reported in Pissarides (2009) and Shimer (2005a). Hence, the average separation rate $\pi^x = 0.036$, job finding rate $q^w = 0.594$ and labor market tightness $\theta = 0.72$ for the periods 1960 to 2004 were targeted. As the evidence for the returns to scale in matching is not persuasively conclusive, I will base the benchmark calibration on the assumption $\xi = 1$. Deviations from the constant returns to scale scenario will then be introduced by setting ξ to the boundary values $\xi \in \{0.5, 1.5\}$. To target the same unemployment rate I will recalibrate the efficiency parameter of the matching function $\mathcal{M}_0$ accordingly. Chetty (2006b) estimates the parameter of relative risk-aversion close to 1. Chetty and Szeidl (2007) argue that for small shocks such as an unemployment spell this value could be considerably bigger. I set the coefficient of risk-aversion to $\sigma = 2$. Nickell et al. (2005) report that a 10% increase in the replacement ratio leads to an increase of unemployment by 1.11%-points. I chose the value for the elasticity of the search cost function ν to be 3. This slightly overestimates the responsiveness reported by Nickell et al. (2005) but implies that the corrected unconditional matching elasticity is not unreasonably large. Appendix section B explains this correction procedure. The correction has to be made because search intensity is usually unobserved which implies that estimates of the matching function elasticities take the endogenous adjustment of search intensity into account which is not done by the deep parameter η. Taking an estimate of $\tilde{\eta} = 0.4$ for the United States which comes close to the values reported by Anderson and Burgess (2000) implies that $\eta = \tilde{\eta}\frac{\nu}{\nu-1} = 0.6$.[37] The bargaining power is then set accordingly to $\omega = 0.6$

[37] Note that the correction formula was derived from the optimal job search condition of risk-neutral workers while the benchmark calibration is done for the case of risk-aversion. The correction is therefore

to rule out any additional unbalanced search externalities that would add on top to the ones created in the presented non-constant returns to scale scenarios. In the benchmark calibration I rule out decreasing marginal productivity and set the marginal product to $y = a = 1$. In equilibrium this will imply a wage that is only slightly lower than 1. A critical choice is the one of the value of unemployment $z = b + h - k(s)$. Although I will look at optimal UB I need a value for currently implement benefits that lead to the observed unemployment rate. The OECD tax benefit calculator gives an initial replacement rate of approximately 50% to 70% depending on personal characteristics like marital status and number of children, etc. Benefits last for 6 months. Afterwards the unemployed worker receives other social assistance payments. To take this loss of unemployment income into account I target the lower bound of the tax benefit calculator's outcome and set $b = 0.5$. Hall and Milgrom (2008) derive a total value of unemployment of $z = 0.71$ that I will target as well, i.e. $h - s(k) = 0.21$. It is not clear how much weight to put on pure home production h on one side and the effort costs of search $k(s)$ on the other side. Hagedorn and Manovskii (2008) argue that home production might be substantial at the same time $b + h$ must not exceed the wage w because I want to rule out an equilibrium where no search, i.e. $s = 0 \Rightarrow k(0) = 0$, is optimal. I set $h = 0.3$ and $k_0 = 0.05$ to comply with these constraints.

Table 5.1: Benchmark calibration

Parameter	Value	Source/Target
r	0.004	Pissarides (2009)
π^x	0.036	Shimer (2005a)
ν	3	$\epsilon_h^u \approx 1.1$, Nickell et al. (2005)
η	0.6	$\tilde{\eta} = 0.4$, Anderson and Burgess (2000)
ω	0.6	no additional inefficiency
c	0.22	average θ, Pissarides (2009)
b	0.5	OECD tax benefit calculator
h	0.3	$b + h$ almost at w, Hagedorn and Manovskii (2008)
$\mathcal{M}_0$	0.5973	job finding probability, Shimer (2005a)
k_0	0.05	$z = 0.71$, Hall and Milgrom (2008)
σ	2	Chetty (2006b) and Chetty and Szeidl (2007)
ξ	1	returns to scale, benchmark
$y = a$	1	normalization

Table E.1 in the appendix summarizes all calibrations for $b = 0.5$ for different combinations of coefficients of constant relative risk-aversion, returns to scale in the matching function and degrees of decreasing marginal productivity in order to match q^w and consequently u of our benchmark by adjusting $\mathcal{M}_0$ accordingly.[38]

not completely precise but should give a reasonable value for the unconditional elasticity.

[38]This procedure obviously eliminates all effects of different choices of market size L which are absorbed by $\mathcal{M}_0$. For the presented values of $\mathcal{M}_0$ a local labor market size of $L = 1000$ was assumed.

5.2 Numerical results

To study the effects over the business cycle I used shocks to factor productivity a of plus and minus 10%, which are clearly at the upper bound of reasonable values. Table 5.2 states optimal UB for different combinations of productivity shocks and assumptions concerning returns to matching and relative risk-aversion.

Table 5.2: Optimal unemployment benefits over the cycle

σ	ξ	a		
		0.9	1.0	1.1
	0.5	0.0407	0.0437	0.0465
0	1.0	0.0000	0.0000	0.0000
	1.5	-0.0136	-0.0145	-0.0153
	0.5	0.3338	0.3921	0.4515
2	1.0	0.3213	0.3785	0.4369
	1.5	0.3171	0.3740	0.4320

Three important findings can be inferred from those results. First, the level of optimal Pigouvian UB to correct for excessive or too little aggregate search is rather small. For the boundary values of reasonable choices of ξ I find that for decreasing returns to scale benefits have to be set to about 4.4% of the wage to correct for excessive search while for increasing returns UB have be to -1.5% of the wage. Second, the responsiveness of optimal Pigouvian UB over the cycle is very mild given the size of the productivity shocks. From the worst to the best productivity state UB vary from 0.0407 to 0.0465 if returns are decreasing and from -0.0136 to -0.0153 if returns are increasing. And third, in case of risk-aversion one can establish a strong pro-cyclicality of UB due to the within-period insurance motive which overturns the counter-cyclicality due to search correction in the case of increasing returns to matching. I will comment on all three findings in the rest of this section.

The simulation results show that the variation of optimal b over the cycle is quantitatively not very important. But this result might be a direct consequence of the Shimer (2005a)-puzzle. Recall that optimal b has to move with Γ^S, the social per-period surplus, as restated in (5.15) for the case of risk-neutral workers.

$$b = (1 - \xi)\Gamma^S(1 - u). \tag{5.15}$$

Shimer argues that the canonical model cannot reproduce the comparably large elasticity of labor market tightness θ w.r.t. labor productivity shocks a that is observed in the data. While the observed elasticity is $\epsilon_a^\theta = 7.56$ the model only predicts $\epsilon_a^\theta = 1.71$ for an otherwise reasonable calibration. Expressing (5.15) in terms of elasticities[39] gives

$$\epsilon_a^b = \left[(1-\tilde{\eta}_v) + \frac{\tilde{\eta}_v u}{1+(\xi-1)(1-u)}\right] \cdot \epsilon_a^\theta. \tag{5.16}$$

Clearly, if ϵ_a^θ would be blown up by the factor 4.42 to match the data also optimal benefits should react stronger to the business cycle by the same factor. Obviously, if one thinks that the Shimer-Puzzle stems from a misspecification of the model it has to be pointed out that the same model was used to derive the optimal benefit formula. However, if the low sensitivity merely results from an unrealistic calibration, as argued by Hagedorn and Manovskii (2008), then (5.16) is correctly specified and the sensitivity of optimal UB to the cycle can be amplified by e.g. increasing the value of home production. This would have two consequences in the model. On one hand, wages become less flexible which boosts the variability of the match surplus and hence would imply more variation in optimal UB over the cycle. On the other hand, the surplus as such becomes smaller and so does the level of optimal UB. Hence, changing the calibration as suggested by Hagedorn and Manovskii (2008) would trade-off my findings one and two.

Finding three stems from the strong pro-cyclicality requirement for the benefits to insure the workers against unemployment risk within a period. The reason is that wages strongly co-move with the cycle which would in good times increase the difference of income from employment versus income from unemployment if latter was constant. Due to their sticky wage assumption this effect is less pronounced in Landais et al. (2010). Observe that in the current set-up intertemporal insurance and consumption smoothing between periods is ignored as I just compare steady states with different levels of aggregate productivity and assume that government budget has to be balanced period by period. Introducing a motive for intertemporal insurance would require a full dynamic, stochastic framework beyond the scope of this paper which tries to focus on the Pigouvian character of UB rather then the provision of insurance. Assuming that the government - in contrast to the workers - faces an *intertemporal* budget constraint that allows to shift resources over time it would mimic precautionary savings of the workers to smooth their consumption over the cycle. Hence, intertemporal insurance would demand counter-cyclical benefits. This might counteract the strong pro-cyclicality found for within period insurance.

So far I assumed that marginal productivity is independent of the level of employment. Relaxing this assumption as done in Landais et al. (2010) introduces a stronger sensitivity of the model to changes in the parameterization that requires some recalibration effort. However, it does not imply an amplification of the surplus sensitivity as demonstrated by

[39]The derivation is provided in appendix A.

the results in table E.2. The intuition is straightforward. A positive shock to productivity directly increases the surplus and therefore employment. But the increase in employment reduces the marginal product and therefore dampens the total effect. Hence, in contrast to Landais et al. (2010) the degree of decreasing marginal productivity interacts with the extent of the search externality in a negative way.

6 Conclusion

Beside the function of insuring workers against the risk of unemployment, unemployment benefits might also be used as a Pigouvian instrument. When the magnitude of distorted aggregate search depends on the state of the economy along the business cycle, unemployment benefits optimally have to vary over the cycle too, even if workers are risk-neutral. The paper presents a Diamond-Mortensen-Pissarides model that explicitly allows for non-constant returns to matching that generate this type of externalities as the additional level effects of unemployment on the match efficiency are not taken into account by the agents. The presented model nests the model of Landais et al. (2010) who find that unemployment benefits should always be positive and even more so in bad times even if the insurance provision motive is neglected. It is shown that their result is sensitive to the choice of the welfare criterion. After taking aggregate profits into account and dropping the criticized assumption of wage rigidity for new hires, I derive a generalized Hosios-condition guaranteeing constrained efficiency. If returns to scale are non-constant another dimension of search externalities related to the aggregate amount of search effort is introduced that requires government intervention in any case. The implementation of the optimal allocation involves pro-cyclical (counter-cyclical) benefits correcting for excessive (too little) aggregate search activity if the matching function exhibits decreasing (increasing) returns to scale. In a numerical exercise I introduce risk-aversion to assess the role of Pigouvian benefits compared to the typical function of insurance provision. It is shown that the quantitative role is rather limited. However, this conclusion might be premature in light of the fact that the low variability of the optimal unemployment benefits is directly connected to the implausibly low responsiveness of the match surplus to productivity shocks for this class of models, an enigma known as the Shimer-puzzle.

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A Derivations and proofs

Derivations of the elasticities in section 2.1. Note that $\frac{\partial \Phi(Lu \cdot m(s,\theta))}{\partial Lu \cdot m(s,\theta)} = \frac{\Phi(Lu \cdot m(s,\theta))}{Lu \cdot m(s,\theta)} \cdot \xi$ and $\frac{\partial m(s,\theta)}{\partial \theta} = \frac{m(s,\theta)}{\theta} \cdot (1-\eta)$.

$$
\begin{aligned}
\frac{\partial q^w(s,u,\theta)}{\partial \theta} &= \frac{1}{Lu} \cdot \frac{\partial \Phi(Lu \cdot m(s,\theta))}{\partial Lu \cdot m(s,\theta)} \cdot Lu \cdot \frac{\partial m(s,\theta)}{\partial \theta} \\
&= \frac{\Phi(Lu \cdot m(s,\theta))}{Lu \cdot m(s,\theta)} \cdot \xi \cdot \frac{m(s,\theta)}{\theta}(1-\eta) \\
&= (1-\eta)\xi \frac{q^w}{\theta} = (1-\eta)\xi q^f = \eta_v q^f.
\end{aligned}
$$

Note that $\Phi_u \equiv \frac{\partial \Phi(Lu \cdot m(s,\theta))}{\partial u} = \xi q^w L$ and $\Phi_L \equiv \frac{\partial \Phi(Lu \cdot m(s,\theta))}{\partial L} = \xi q^w u$.

$$
\begin{aligned}
\frac{\partial q^w(s,u,\theta)}{\partial u} &= \frac{\Phi' \cdot Lu - L \cdot \mathcal{M}}{L^2u^2} = \frac{\xi q^w L \cdot Lu - L \cdot \mathcal{M}}{L^2u^2} \\
&= (\xi - 1) q^w u^{-1} \\
\frac{\partial q^w(s,u,\theta)}{\partial L} &= (\xi - 1) q^w L^{-1}.
\end{aligned}
$$

■

Proof of proposition 4.3. The optimal UB formula given by (4.12) is derived as follows. First, compare (2.26) and (4.6). Given that $\omega = \eta$ and $t = 0$ efficiency is restored if and only if $\xi S^S = S$ or $\xi \Gamma^S = \Gamma$. Use the relation of both per-period surpluses $\Gamma = \Gamma^S - b/(1-u)$ and solve for b. To prove the last statement of proposition 4.3 one simply has to show that Γ^S and $1-u$ are pro-cyclical. First, note the following relation of the social and the decentralized per-period surplus

$$
\Gamma^S = S^S(r+\pi^x) = S(r+\pi^x) + \frac{b}{1-u} \equiv \Gamma + \frac{b}{1-u}. \tag{A.1}
$$

Insert the optimal benefits (4.12) in the job creation condition $(1-\omega)\frac{\Gamma}{r+\pi^x} = \frac{c}{q^f}$ to get

$$
(1-\omega)\xi \frac{\Gamma^S}{r+\pi^x} = \frac{c}{q^f}. \tag{A.2}
$$

Next, eliminate search intensity s by inserting for $k(s) - sk'(s) = (\nu - 1)k(s) = \frac{\nu-1}{\nu}\frac{\omega}{1-\omega}c\theta$ and use the unconditional probability[40] $\tilde{q}^f$. Rearrange to get

$$
y - h = \frac{r+\pi^x}{(1-\omega)\xi}\frac{c}{\tilde{q}^f} + \frac{\nu-1}{\nu}\frac{\omega}{1-\omega}c\theta. \tag{A.3}
$$

[40] See appendix section B.

Clearly, when a increases y and consequently the left-hand side have to rise. As the right-hand side is increasing in θ[41], I have established that $d\theta/da > 0$. Applying this relation to (A.2) and the Beveridge curve using unconditional job finding probabilities implies that $d\Gamma^S/da > 0$ and $d(1-u)/da > 0$ which completes the proof. ■

Derivations of the elasticity in section 5. First take the total differential of (4.12).

$$db = (1-\xi)(1-u)\, d\Gamma^S + (1-\xi)\Gamma^S\, d(1-u) \quad \Leftrightarrow \quad \epsilon_a^b = \epsilon_a^{\Gamma^S} + \epsilon_a^{1-u}. \tag{A.4}$$

I will rewrite (A.4) in terms of ϵ_a^θ but before that I will contrast the social per-period surplus Γ^S with the decentralized per-period surplus Γ

$$\Gamma \equiv [y - (h - b/(1-u) - k(s)) - sk'(s)] \quad \text{such that} \quad \Gamma^S - \frac{b}{1-u} = \Gamma. \tag{A.5}$$

Inserting the optimal benefits gives $\xi\Gamma^S = \Gamma$, i.e. at the optimum benefits will not only force the decentralized per-period surplus to coincide with the optimal share of the social per-period surplus but also both surpluses will move over the cycle in a synchronized way, i.e. $\epsilon_a^{\Gamma^S} = \epsilon_a^\Gamma$. Rewrite the job creation condition, take the total differential and rearrange to get

$$\epsilon_a^\Gamma = (1-\tilde{\eta}_v)\epsilon_a^\theta = \epsilon_a^{\Gamma^S}, \tag{A.6}$$

where I used the unconditional vacancy filling probability $\tilde{q}^f$[42]. Next, one has to transform ϵ_a^{1-u} which is equal to $\epsilon_\theta^{1-u} \cdot \epsilon_a^\theta$. The derivative of $1-u = \frac{\tilde{q}^w(u,\theta)}{\pi^x + \tilde{q}^w(u,\theta)}$ w.r.t. a can be written as

$$\frac{d(1-u)}{da} = \frac{\partial(1-u)}{\partial \tilde{q}^w} \cdot \frac{\partial \tilde{q}^w}{\partial \theta} \cdot \frac{d\theta}{da} - \frac{\partial(1-u)}{\partial \tilde{q}^w} \cdot \frac{\partial \tilde{q}^w}{\partial u} \cdot \frac{d(1-u)}{da}, \tag{A.7}$$

where I used the fact that $\frac{du}{da} = -\frac{d(1-u)}{da}$. The first term in (A.7) can be computed as follows

$$\begin{aligned}
\frac{\partial(1-u)}{\partial\theta} &= \frac{\tilde{\eta}_v\theta^{-1}\tilde{q}^w\,[\pi^x + \tilde{q}^w]}{[\pi^x + \tilde{q}^w]^2} - \frac{\tilde{\eta}_v\theta^{-1}\,(\tilde{q}^w)^2}{[\pi^x + \tilde{q}^w]^2} \\
&= (1-u)\tilde{\eta}_v\theta^{-1}\,[u + 1 - u] - \tilde{\eta}_v\theta^{-1}(1-u)^2 \\
&= \tilde{\eta}_v\theta^{-1}(1-u)u.
\end{aligned} \tag{A.8}$$

Proceed analogously to get $\frac{\partial(1-u)}{\partial u} = (\xi - 1)(1-u)$. Both can now be combined to get

$$\epsilon_a^{1-u} = \frac{\tilde{\eta}_v u}{1 + (\xi-1)(1-u)} \cdot \epsilon_a^\theta. \tag{A.9}$$

[41] See appendix C for more details concerning this statement.
[42] See appendix section B.

In total the elasticity of optimal b w.r.t. a is therefore given by

$$\epsilon_a^b = \left[(1-\tilde{\eta}_v) + \frac{\tilde{\eta}_v u}{1+(\xi-1)(1-u)}\right] \cdot \epsilon_a^\theta. \tag{A.10}$$

■

Proof of proposition 4.2. I introduce proportional taxes/subsidies for all flow variables: output, value of leisure, vacancy posting costs, the worker's and the employer's wage rate. Denote the corresponding proportional tax rates t^y, t^z, t^c, t^w and t^f. First, assume that our surplus sharing condition (2.22) is unaffected. Rearrange to get $\omega rJ - (1-\omega)rW = -(1-\omega)rU$ and insert to get

$$w = \frac{\omega(1-t^y)y + (1-\omega)(1-t^z)z + \omega(1-t^c)c\theta}{\omega(1-t^f) + (1-\omega)(1-t^w)}. \tag{A.11}$$

Inserting in the job creation condition with taxes,

$$\frac{(1-t^y)y - (1-t^f)w}{r+\pi^x} = (1-t^c)\frac{c\theta}{q^w}, \tag{A.12}$$

and analyzing the condition for optimal search with taxes,

$$(1-t^c)c\theta\frac{\omega}{1-\omega} = (1-t^z)sk'(s), \tag{A.13}$$

reveals that for no values of our taxes one can mimic (2.26). Now consider the case where the sharing rule is affected by taxation because I explicitly derive it from Nash bargaining. This implies

$$\omega(1-t^w)rJ - (1-\omega)(1-t^f)rW = -(1-\omega)(1-t^f)rU. \tag{A.14}$$

The wage would then be given by

$$w = \frac{\omega}{1-t^f}\left[(1-t^y)y + (1-t^c)c\theta\right] + \frac{1-\omega}{1-t^w}(1-t^z)z. \tag{A.15}$$

Again, after inserting in (A.12) it is easy to verify that there is no way to implement (2.26). ■

B Conditional versus unconditional matching function

The model treats search intensity as an endogenous variable which enters the matching function and itself is a function of u and v. As search intensity is very hard to measure, empirical studies typically only estimate elasticities w.r.t. u and v neglecting search intensity. Those estimates then take endogenous responses of s implicitly into account and therefore do not coincide with the their theoretical counterparts η_u and η_v that measure changes in the number of matches for constant search intensity. Putting a little bit of structure on the search cost function will allow to correct the estimated elasticities and relate them to the deep parameters of the model. As an illustration, this is only done for the case of risk-neutrality as the optimal job search condition simplifies considerably in that case. Assume that the function $k(\cdot)$ is characterized by a constant elasticity $\epsilon_s^k = \nu$. Consequently, also the function $\kappa(s) \equiv sk'(s)$ will have a constant elasticity $\epsilon_s^\kappa = \nu$. Rewrite the optimal job search condition (3.7) as

$$s = \kappa^{-1}\left(\frac{\omega}{1-\omega}c\theta\right) \equiv s(\theta). \tag{B.1}$$

This implies the following elasticities $\epsilon_\theta^{s(\theta)} = 1/\nu$ and $\epsilon_\theta^{k(s(\theta))} = 1$. Now eliminate s by inserting (B.1) in the function for aggregate search activity $Lm(su, v)$ *conditional* on s. This results in an *unconditional* function $L\tilde{m}(u, v)$ with the following elasticities

$$\epsilon_u^{\tilde{m}} = \eta - \frac{\eta}{\nu} \equiv \tilde{\eta} \quad \text{and} \quad \epsilon_v^{\tilde{m}} = 1 - \eta + \frac{\eta}{\nu} \equiv 1 - \tilde{\eta}. \tag{B.2}$$

Clearly, also the unconditional aggregate search activity function has constant returns to scale. The elimination of s just implies a re-weighting of the elasticities in favor of the vacancy rate. Further, the total matching elasticities are simply $\epsilon_u^{\tilde{\mathcal{M}}} = \tilde{\eta} \cdot \xi \equiv \tilde{\eta}_u$ and $\epsilon_v^{\tilde{\mathcal{M}}} = (1-\tilde{\eta}) \cdot \xi \equiv \tilde{\eta}_v$. Thus the degree of returns to scale is independent of whether one uses the conditional or the unconditional matching function. This is obviously a direct consequence of the invariance of the labor market tightness θ and consequently search intensity s in (3.7) to changes in the market size. The derived reduced-form matching function not only allows to work with a smaller dimensional model but also helps to relate the model's conditional elasticities to the observed ones: $\tilde{\eta}_u$ and $\tilde{\eta}_v$. Note however that the optimality conditions are obviously unaffected and are still given by $\omega^w = \eta_u$ and $\omega^f = \eta_v$. The next proposition states the generalized Hosios-condition in terms of the observed, unconditional elasticities.

Proposition B.1. ***Generalized Hosios-condition for unconditional elasticities:*** *The allocation is efficient if, in addition to $b = 0$, the effective bargaining powers coincide* *with corrected versions of the unconditional elasticities, i.e. if*

$$\omega^w = \tilde{\eta}_u + \frac{\xi}{\nu} \qquad \text{and} \qquad \omega^f = \tilde{\eta}_v - \frac{\xi}{\nu}. \tag{B.3}$$

Proof. This follows directly from combining the derived relation of the conditional and unconditional elasticities and proposition 4.1. ∎

Importantly, one must strictly distinguish the unconditional matching function in case of variable search intensity from the case with fixed search effort often used in the literature where optimality is given by $\omega^w = \tilde{\eta}_u$ instead of (B.3). Observe how the fixed search effort case is nested in this model by letting $\nu \to \infty$.

C Uniqueness and multiplicity of equilibria

This section derives conditions for uniqueness and multiplicity of equilibria for the case of risk-neutrality. I will concentrate on the non-policy case, i.e. $b = 0$ and $T = 0$ to isolate multiplicity stemming from the matching technology. First, let the implicit solution to the Beveridge curve (2.8) be $u(\theta)$. If $\xi = 1$, then the solution for u can be expressed explicitly. Take the total differential of the rearranged Beveridge curve, $uq^w(s(\theta), u, \theta) + u\pi^x = \pi^x$, to arrive at

$$\begin{aligned}
& u\frac{\partial q^w}{\partial s(\theta)}\frac{\partial s(\theta)}{\partial \theta}\, d\theta + u\frac{\partial q^w}{\partial \theta}\, d\theta + u\frac{\partial q^w}{\partial u}\, du + q^w\, du + \pi^x\, du = 0 \\
\Leftrightarrow\ & u\frac{q^w}{s(\theta)}\frac{s(\theta)}{\theta}\epsilon_s^{q^w}\epsilon_\theta^{s(\theta)}\, d\theta + u\frac{q^w}{\theta}\epsilon_\theta^{q^w}\, d\theta + u\frac{q^w}{u}\epsilon_u^{q^w}\, du + q^w\, du + \pi^x\, du = 0 \\
\Leftrightarrow\ & \frac{q^w}{\theta}\left[\frac{\eta_u}{\nu} + \eta_v\right] d\theta + \frac{q^w}{u}\left[\xi - 1 + \frac{1}{1-u}\right] du = 0
\end{aligned}$$

Clearly, for all degrees of scale it is always true that $\epsilon_\theta^{u(\theta)} < 0$. For the job search condition (3.7) I work with the simple iso-elastic functional form of the search effort function, $k(s) = k_0 s^\nu$, which is also used in the simulation part. The explicit solution of the job search condition is then

$$s = \left[\frac{\omega}{1-\omega}\frac{c}{\nu k_0}\right]^{\frac{1}{\nu}} \theta^{\frac{1}{\nu}}. \tag{C.1}$$

as derived in appendix section B. Therefore the following relations can be established

$$k(s(\theta)) = \frac{\omega}{1-\omega}\frac{c}{\nu}\theta, \tag{C.2}$$

$$k(s(\theta)) - s(\theta)k'(s(\theta)) = (1-\nu)k(s(\theta)) = \frac{1-\nu}{\nu}\frac{\omega}{1-\omega}c\theta < 0. \tag{C.3}$$

Insert the last expression, $s(\theta)$ and $u(\theta)$ in the job creation condition (2.14) and rearrange to get

$$\omega^f\left[\frac{y-h}{r+\pi^x}\right]=\frac{c}{q^f(\theta)}+\frac{\omega^f}{r+\pi^x}\frac{\nu-1}{\nu}\frac{\omega}{1-\omega}c\theta. \tag{C.4}$$

where $q^f(\theta)\equiv q^f(s(\theta),u(\theta),\theta)$. Every θ^* that solves (C.4) gives an equilibrium.

Lemma C.1. $\frac{dq^f(\theta)}{d\theta}>(<)\ 0$ *is a sufficient (necessary) condition for uniqueness (multiplicity) of equilibria.*

Proof. Simply observe that $\frac{dq^f(\theta)}{d\theta}>(<)\ 0$ is a sufficient (necessary) condition for the right-hand side of (C.4) to monotonically decrease (to be non-monotonic) in θ. ∎

I will now take a closer look at $q^f(\theta)$. The elasticity of $q^f(\theta)$ w.r.t. θ is given by

$$\epsilon_\theta^{q^f(\theta)}=\eta_v-1+\frac{\eta_u}{\nu}+(\xi-1)\epsilon_\theta^{u(\theta)}, \tag{C.5}$$

where $\epsilon_\theta^{u(\theta)}<0$ as derived before. Observe that this elasticity is negative if and only if the following condition holds

$$\eta\frac{1-\nu}{\nu}+\frac{\xi-1}{\xi}\left[1+\epsilon_\theta^{u(\theta)}\right]<0. \tag{C.6}$$

Proposition C.1. *Assume that* $|\epsilon_\theta^{u(\theta)}|<1$ *then* $\xi\leq(>)\ 1$ *is a sufficient (necessary) condition for uniqueness (multiplicity) of equilibria.*

Proof. This follows directly from equation (C.6). ∎

The assumption of $|\epsilon_\theta^{u(\theta)}|<1$ does not seem to be unreasonable. In my simulation, using $b=T=\sigma=0$, this elasticity is -0.57 for the calibration with $\xi=0.5$ and -0.59 for $\xi=1.5$.[43] Hence, multiple equilibria can only occur with increasing returns to scale of the matching function.

D Implementation by wage dependent benefits

Often UB are designed in a way such that they are a constant fraction of the wage, i.e. as a replacement ratio. This section shows how the findings would change in this case. Contrary to absolute UB the optimal replacement ratio should be pro-cyclical (counter-cyclical) if $\xi>1$ ($\xi<1$). As common in the policy oriented strand of the macro-labor literature I assume that UB are given by $b=\rho w$, where ρ denotes the replacement ratio. While this seems to be an appropriate specification in many contexts it is problematic for

[43]Inserting this elasticity and the other parameters in (C.6) reveals that multiplicity is not an issue in my simulations.

the exercise of looking at comparative statics w.r.t. productivity. The reason is that in reality b depends on the previous wage and not on a currently by the business cycle affected wage index[44] as suggested in this specification. Hence, the original specification taking b constant seems to be more appropriate. Nevertheless, I will provide a short analysis of wage dependent benefits. A single bargaining pair takes the wage index as given, i.e. it does not change the sharing rule if it was derived from explicit Nash-bargaining. Hence, one can simply insert $z = h + \rho w - k(s)$ into (2.23) and solve for the wage

$$w = \frac{(1-\omega)\left[h - k(s) + sk'(s)\right] + \omega y}{1-(1-\omega)\rho}. \tag{D.1}$$

I have established that optimal benefits in absolute numbers should be proportional to the social surplus and the employment rate. If benefits vary with current wages and wages fluctuate stronger than the match surplus and $1-u$ than the derived optimal business cycle responses will be reversed. Insert $b = \rho w$ in (4.12) to get

$$\rho = (1-\xi)\frac{\Gamma(1-u)}{w}. \tag{D.2}$$

Recall that given the assumptions on the functional form of $k(\cdot)$ one can rewrite $k(s) - sk'(s) = (1-\nu)k(s)$. Insert (D.1) in (D.2) to get

$$\frac{\rho}{1-(1-\omega)\rho} = (1-\xi)\left[\frac{y - h - (\nu-1)k(s)}{\omega y + (1-\omega)\left[h + (\nu-1)k(s)\right]}\right]. \tag{D.3}$$

The right-hand side is increasing in ρ. It seems that whether or not the right-hand side increases with a will depend on the bargaining power ω. Clearly, for $\omega \to 1$ the right-hand side is decreasing in a as $ds/da > 0$. For the other case $\omega \to 0$ it is analytically ambiguous but numerical simulations suggest that the relevant term $\frac{y}{h+(\nu-1)k(s)}$ is quite robustly decreasing in a as well. Hence, wages fluctuate stronger than the social surplus and the cyclical pattern of the optimal replacement ratio is reversed compared to UB in absolute terms.

E Tables

[44] The average wage or wage index coincides with individual wages of the employed workers as workers are homogeneous in the present framework.

Table E.1: Different calibration scenarios

Scenario	ξ	σ	α	$\mathcal{M}_0$	w	s	u	θ	q^w	T
Benchmark	1	2	1.000	0.5973	0.9894	1.2348	0.0571	0.7188	0.5940	0.0303
Case 2	0.5	2	1.000	4.5025	0.9894	1.2348	0.0571	0.7188	0.5940	0.0303
Case 3	1.5	2	1.000	0.0792	0.9894	1.2348	0.0571	0.7188	0.5940	0.0303
Case 4	1	0	1.000	0.6509	0.9902	1.1325	0.0571	0.6602	0.5940	0.0303
Case 5	0.5	0	1.000	4.7001	0.9902	1.1325	0.0571	0.6602	0.5940	0.0303
Case 6	1.5	0	1.000	0.0901	0.9902	1.1325	0.0571	0.6602	0.5940	0.0303
Case 7	1	0	0.995	0.7595	0.9539	1.0394	0.0571	0.5104	0.5940	0.0303
Case 8	0.5	0	0.995	5.0775	0.9539	1.0394	0.0571	0.5104	0.5940	0.0303
Case 9	1.5	0	0.995	0.1136	0.9539	1.0394	0.0571	0.5104	0.5940	0.0303

Table E.2: Optimal unemployment benefits for different marginal product assumptions

$\alpha = 1.000$					$\alpha = 0.995$				
ξ		a			ξ		a		
		0.9	1.0	1.1			0.9	1.0	1.1
0.5	b	0.0407	0.0437	0.0465	0.5	b	0.0345	0.0371	0.0394
	Γ^S	0.0837	0.0897	0.0952		Γ^S	0.0707	0.0759	0.0805
	u	0.0284	0.0258	0.0237		u	0.0251	0.0228	0.0210
1.0	b	0.0000	0.0000	0.0000	1.0	b	0.0000	0.0000	0.0000
	Γ^S	0.0419	0.0447	0.0472		Γ^S	0.0352	0.0375	0.0397
	u	0.0263	0.0240	0.0221		u	0.0233	0.0212	0.0195
1.5	b	-0.0136	-0.0145	-0.0153	1.5	b	-0.0114	-0.0122	-0.0129
	Γ^S	0.0279	0.0297	0.0314		Γ^S	0.0234	0.0249	0.0263
	u	0.0257	0.0234	0.0216		u	0.0228	0.0207	0.0191